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A FINITE-DIFFERENCE PROGRAM  
FOR STRESSES IN ANISOTROPIC,  
LAYERED PLATES IN BENDING

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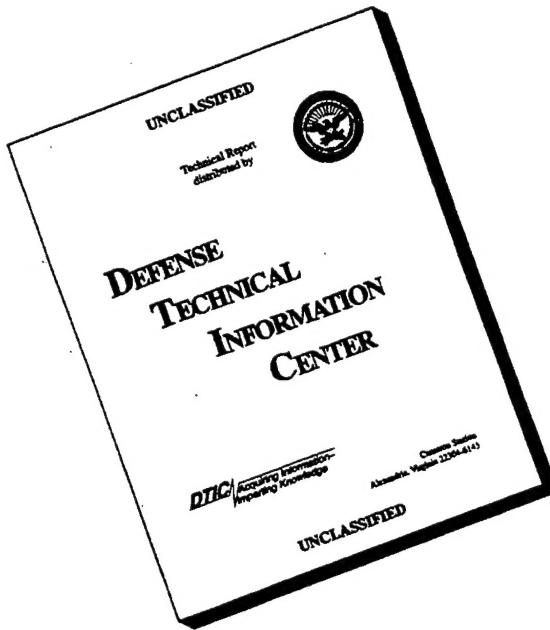
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16. ABSTRACT  Results from the initial phase of a study of the interlaminar stresses induced in a layered laminate that is bent into a cylindrical surface are given. The laminate is modeled as a continuum, and the resulting elasticity equations are solved using the finite-difference method. The report sets forth the mathematical framework, presents some preliminary results, and provides a listing and explanation of the computer program. Significant among the results are apparent symmetry relationships that will reduce the numerical size of certain problems and an interlaminar stress behavior having a sharp rise at the free edges.			
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## LIST OF SYMBOLS

<u>Symbol</u>	<u>Definition</u>
A	laminate configuration; coefficient matrix [equation (22)]
B	laminate configuration; load vector [equation (22)]
$B'_{ij}$	constitutive matrix (Appendix A)
$B_u, B_v$	laminate load constants [equation (7)]
$C_i$	laminate load constants [equation (5)]
$c'_{ij}$	elastic coefficients with respect to $x', y', z'$
$c_{ij}$	elastic coefficients with respect to $x, y, z$ [equation (1)]
C,D	load values [equation (33)]
$D_v$	laminate load constant [equation (7)]
$D'_{ij}$	constitutive matrix (Appendix A)
$E_{ii}$	Young's moduli
$G_{ij}$	shear moduli
$h_i$	node spacing (Fig. 2)
I,J	nodal coordinates (Figs. 2 and 3)
$M, M_i$	applied moments [equation (4a)]
m	layer number (Fig. 1)
U,V,W	displacement functions [equation (6)]
u,v,w	displacements with respect to $x, y, z$ [equations (3) and (8)]
x,y,z	laminate coordinate axes (Fig. 1)
$x', y', z'$	lamina orthotropic axes (Fig. 1)

---

## LIST OF SYMBOLS (Concluded)

<u>Symbol</u>	<u>Definition</u>
X	unknown vector [equation (22)]
$\gamma_{ij}$	shear strains [equation (2)]
$\epsilon_i$	normal strains [equation (2)]
$\theta$	lamina orientation angle (Fig. 1)
$\sigma_i$	normal stress [equation (1)]
$\tau_{ij}$	shear stress [equation (1)]
$\nu_{ij}$	Poisson's ratio

Symbols appearing in the computer program are defined in the subsection entitled "The Mesh Simulation."

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## A FINITE-DIFFERENCE PROGRAM FOR STRESSES IN ANISOTROPIC, LAYERED PLATES IN BENDING

### INTRODUCTION

One critical feature associated with structural composites of laminated construction, using materials or geometrical arrangements that exhibit different elastic properties from layer to layer, is the possibility that the glued layers will separate or delaminate. This was undoubtedly realized from the outset of their use, and a brief historical sketch of the American scene is presented by Pipes [1]. However, the earliest serious investigation into the cause of delamination-type failure, namely the interlaminar stress problem, was apparently done in Japan by Hayashi [2,3], who reported that the maximum interlaminar shearing stresses occurred at the free edge of a laminate under tension. Hayashi used a plane stress model for the layers and approximated the interlaminar shears by a strain-averaging technique. Using a similar model, Puppo and Evensen [4] likewise discovered a sharp rise in the interlaminar stresses near a free edge. Notably, the use of the above models ignored the interlaminar normal stress. In two publications, Pipes and Pagano [5,6] developed a finite-difference program to solve the exact elasticity equations for a long laminate in uniaxial extension. In their development, the stresses are assumed independent of the axial coordinate and include all six components. The results of this investigation show that a sharp rise in both the interlaminar shear stresses and the normal stress occurs near the free edge. Thereafter, Oplinger [7] did an analysis of angle ply laminates in tension using a model similar to that of References 2 through 4. His approach allows a large number of layers to be considered. Indeed it was discovered that a singularity in the interlaminar shear occurs at the free edge of a laminate of one particular type of construction. An alternative solution to that employed in the above references is used by Rybicki [8] who applied a three-dimensional finite element formulation. His results agree with References 5 and 6.

The present report marks the initial phase of a study of the interlaminar stresses induced in a layered laminate by bending. Following the approach used by Pipes [5], the laminate is modeled as a continuum and the resulting elasticity equations are solved using the finite-difference method. This solution technique is made possible by assuming that the laminate is bent into a cylindrical surface such that the stresses are independent of the axial coordinate. The objective of this report is to set forth the mathematical framework, present some preliminary results, and to avail the computer program to others. The results reveal a simplifying symmetry relationship in the displacements that will allow significant reduction in the size of certain numerical problems. In addition, trends in the interlaminar stress distribution are somewhat similar to those found for stretching problems, in that a sharp rise occurs at the free edge.

## PROBLEM FORMULATION

### Laminate Description

The laminated composites considered in this report consist of rectangular laminae symmetrically stacked with respect to a midplane and bonded together to form a flat laminate. The bonding is assumed to provide perfect adhesion between the laminae, which nullifies the possibility of slip between adjacent laminae thus establishing the conditions of continuous displacements and tractions at each interface. Each individual lamina is considered to be elastic, homogeneous, and orthotropic (i.e., each lamina possesses three planes of elastic symmetry). The assumption of homogeneity eliminates micromechanical effects such as those involving fibers or matrix. The geometry of a typical lamina and laminate is illustrated in Figure 1. One may note that the orthotropic coordinate axes ( $x', y', z$ ) of a lamina are referred through a clockwise rotation about  $z$  to the fixed coordinate axes ( $x, y, z$ ) of the laminate. The laminae are stacked along  $z$  to form a laminate whose sides are normal to  $x, y$ , and  $z$ . Each lamina is given a layer number  $m$ .

Limiting the analysis to linear elastic materials, the constitutive relation for each lamina referred to the  $x, y, z$  coordinate system is

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & c_{16} \\ & c_{22} & c_{23} & 0 & 0 & c_{26} \\ & & c_{33} & 0 & 0 & c_{36} \\ (\text{symmetric}) & & & c_{44} & c_{45} & 0 \\ & & & & c_{55} & 0 \\ & & & & & c_{66} \end{bmatrix} \begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_z \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{bmatrix}, \quad (1)$$

where the elastic constants  $c_{ij}$  are related to the nine orthotropic constants  $c'_{ij}$  through the well known transformation equations of References 9 and 10.<sup>1</sup> By associating the displacements  $u, v$ , and  $w$  with  $x, y$ , and  $z$ , respectively, the strains for each lamina are defined as

---

1. In using the transformation equations in References 9 and 10 substitute  $-\theta$  for  $+\theta$  since here the constants are referred to the unprimed coordinate axes of the laminate.

$$\epsilon_x^m = u_{,x}^m \quad \epsilon_y^m = v_{,y}^m \quad \epsilon_z^m = w_{,z}^m$$

$$\gamma_{yz}^m = w_{,y}^m + v_{,z}^m \quad \gamma_{xz}^m = w_{,x}^m + u_{,z}^m \quad \gamma_{xy}^m = v_{,x}^m + u_{,y}^m , \quad (2)$$

where the comma denotes partial differentiation.

### Loading and Field Quantities

Consider a laminate loaded by bending about  $y$  at the ends  $x = \text{constant}$ . Assuming that the laminate is long enough in the  $x$ -direction and that Saint-Venant's principle holds for a laminate, the resulting stress distribution will be independent of  $x$  in regions sufficiently removed from the areas of loading. Using this assumption and following Lekhnitskii [11], the elastic strain-stress relations can be integrated to yield displacements for each lamina of the form

$$u^m = (C_1 y + C_2 z + C_3) x + U^m(y, z)$$

$$v^m = -\frac{1}{2} C_1 x^2 + C_4 xz + V^m(y, z)$$

$$w^m = -\frac{1}{2} C_2 x^2 - C_4 xy + W^m(y, z) , \quad (3)$$

where  $U^m$ ,  $V^m$ , and  $W^m$  are unknown functions of  $y$ ,  $z$ . The layer number,  $m$ , is left off the constants  $C_i$  because it results that each  $C_i$  must be the same for every lamina in order to satisfy the displacement continuity conditions at the interfaces. Thus, the  $C_i$  are found to be properties of the entire laminate. The displacement equations (3) represent the full three-dimensional elasticity solution that holds for all points in the laminate.

To evaluate the  $C_i$ , the scheme is as follows. Since equations (3) hold for all points in the laminate, they must converge to the plane stress solution, which is an exact solution, in the interior region of the laminate. Integrating the relation [10,12]

$$e_i = B'_{ij} M_j + z D'_{ij} M_j ; \quad i, j = 1, 2, 6 \quad (4a)$$

for the case where  $M_1 = -M$  and  $M_2 = M_6 = 0$ , the plane stress displacements are found to be

$$\begin{aligned} u_{ps} &= (-D'_{11}Mz - B'_{11}M)x - B'_{61}My - \frac{1}{2}D'_{16}Myz + f(z) \\ v_{ps} &= -\frac{1}{2}D'_{16}Mxz - (B'_{21}M + D'_{12}Mz)y + g(z) \\ w_{ps} &= \frac{1}{2}D'_{11}Mx^2 + \frac{1}{2}D'_{16}Mxy + \frac{1}{2}D'_{12}My^2 + f^*(x) + g^*(y) \quad , \quad (4b) \end{aligned}$$

where  $B'_{ij}$  and  $D'_{ij}$  are laminate properties defined in Appendix A, and  $M$  is the applied moment. Comparing equations (3) and (4b) leads to the results:

$$\begin{aligned} C_1 &= 0 & C_2 &= -D'_{11}M \\ C_3 &= -B'_{11}M & C_4 &= -\frac{1}{2}D'_{16}M \quad (5) \end{aligned}$$

and

$$\begin{aligned} U^m(y, z) &\rightarrow B_u y + C_4 yz + U^m(y, z) \\ V^m(y, z) &\rightarrow B_v y + D_v yz + V^m(y, z) \\ W^m(y, z) &\rightarrow -\frac{1}{2}D_v y^2 + W^m(y, z) \quad , \quad (6) \end{aligned}$$

where<sup>2</sup>

$$B_u = -B'_{61}M \quad , \quad B_v = -B'_{21}M \quad , \quad \text{and} \quad D_v = -D'_{12}M \quad . \quad (7)$$

---

2. The extended forms (6) for  $U^m$ ,  $V^m$ , and  $W^m$  are not necessary to the solution.

Substituting the results (6) into equations (3) yields displacements of the following functional form for each layer

$$\begin{aligned}
 u^m &= (C_2 z + C_3)x + (B_u + C_4 z)y + U^m(y, z) \\
 v^m &= C_4 xz + (B_v + D_v z)y + V^m(y, z) \\
 w^m &= -\frac{1}{2} C_2 x^2 - C_4 xy - \frac{1}{2} D_v y^2 + W^m(y, z) ,
 \end{aligned} \tag{8}$$

where  $C_i$ ,  $B_i$ , and  $D_v$  are defined by equations (5) and (7). The strains are found by substituting the displacements (8) into the strain relations (2). The stresses then follow directly using the constitutive relation (1).

It is of interest to examine the strain  $\epsilon_x^m$  which is

$$\epsilon_x^m = C_2 z + C_3 . \tag{9}$$

Should the laminate be a balanced composite, i.e., the laminae are symmetrically stacked, according to composition and orientation with respect to the midplane  $z = 0$ , then  $B'_{ij} = 0$  and from equations (5)  $C_3 = 0$ , which results in a case of pure bending. For the opposite case, an unbalanced composite exhibits an extensional strain,  $C_3$ , in bending. Such coupling effects are common to laminated composites.

### Field Equations and Boundary Conditions

In regions sufficiently removed from the load planes, the nonboundary points must satisfy the reduced equilibrium equations

$$\begin{aligned}
 \tau_{xy,y}^m + \tau_{xz,z}^m &= 0 \\
 \sigma_{y,y}^m + \tau_{yz,z}^m &= 0 \\
 \tau_{yz,y}^m + \sigma_{z,z}^m &= 0 ,
 \end{aligned} \tag{10}$$

where the stresses exhibit no x-dependence, which conforms to an earlier assumption. Substituting for the stresses in terms of displacements yields the field equations for each lamina

$$\begin{aligned}
 c_{66}^m U_{yy}^m + c_{55}^m U_{zz}^m + c_{26}^m V_{yy}^m + c_{45}^m V_{zz}^m + (c_{36}^m + c_{45}^m) W_{yz}^m &= 0 \\
 c_{26}^m U_{yy}^m + c_{45}^m U_{zz}^m + c_{22}^m V_{yy}^m + c_{44}^m V_{zz}^m + (c_{23}^m + c_{44}^m) W_{yz}^m &= 0 \\
 (c_{36}^m + c_{45}^m) U_{yz}^m + (c_{23}^m + c_{44}^m) V_{yz}^m + c_{44}^m W_{yy}^m + c_{33}^m W_{zz}^m \\
 &= -(c_{13}^m C_2 + c_{23}^m D_v + 2c_{36}^m C_4) \quad . \tag{11}
 \end{aligned}$$

The boundary conditions on the free surfaces normal to y are

$$\sigma_y^m = \tau_{xy}^m = \tau_{yz}^m = 0 \tag{12}$$

and on the free surfaces normal to z are

$$\sigma_z^m = \tau_{xz}^m = \tau_{yz}^m = 0 \quad . \tag{13}$$

For continuity at the interfaces, the boundary conditions are:

$$\begin{aligned}
 (u^m, v^m, w^m) &= (u^{m+1}, v^{m+1}, w^{m+1}) \\
 \text{and} \quad (\sigma_z^m, \tau_{xz}^m, \tau_{yz}^m) &= (\sigma_z^{m+1}, \tau_{xz}^{m+1}, \tau_{yz}^{m+1}) \quad , \tag{14}
 \end{aligned}$$

respectively.

It is noted that the corner conditions are ambiguous in that there are five possible conditions out of which only three can be employed at any one time. The remaining two may or may not be satisfied by the solution. Thus, combinations may be tried until some satisfying results are achieved.

## FINITE-DIFFERENCE SIMULATION

### Function Representation

The mathematical basis for the finite-difference method is Taylor's Series. Referring to Figure 2, the Taylor Series expansion for a function  $f$  at some point  $y, z$  about the point (or node)  $I, J$  is

$$\begin{aligned}
 f(y, z) = & f(I, J) + yf_y(I, J) + zf_z(I, J) \\
 & + \frac{1}{2} y^2 f_{yy}(I, J) + \frac{1}{2} z^2 f_{zz}(I, J) + yzf_{yz}(I, J) + \dots
 \end{aligned} \quad (15)$$

Thus, for the specific node  $I-1, J$ , the expansion is

$$f(I-1, J) = f(I, J) - h_1 f_y + \frac{1}{2} h_1^2 f_{yy} - \dots \quad (16)$$

Writing similar expansions for the remaining seven points' neighboring the node  $I, J$  and simultaneously solving expansions for the first and second derivatives yields the finite-difference approximations for these derivatives. All but the last of these expressions, given below, are taken from Forsythe and Wasow [13]. They are

$$\begin{aligned}
 f_y(I, J) &= \frac{1}{h_1 + h_2} \left[ \frac{h_1}{h_2} f(I+1, J) - \frac{h_2}{h_1} f(I-1, J) \right] + \frac{h_2 - h_1}{h_1 h_2} f(I, J) + O(h^2) \\
 f_z(I, J) &= \frac{1}{2h_3} \left[ f(I, J+1) - f(I, J-1) \right] + O(h^2) \\
 f_{yy}(I, J) &= \frac{2}{h_1 + h_2} \left[ \frac{1}{h_2} f(I+1, J) + \frac{1}{h_1} f(I-1, J) \right] - \frac{2}{h_1 h_2} f(I, J) + O(h^2) \\
 f_{zz}(I, J) &= \frac{1}{h_3^2} \left[ f(I, J+1) + f(I, J-1) - 2f(I, J) \right] + O(h^2) \\
 f_{yz}(I, J) &= \frac{1}{2h_3(h_1 + h_2)} \left[ f(I+1, J+1) - f(I-1, J+1) - f(I+1, J-1) \right. \\
 &\quad \left. + f(I-1, J-1) \right] + O(h^2) \quad ,
 \end{aligned} \quad (17)$$

where  $h$  is an order of magnitude equal to  $h_1$ ,  $h_2$ , or  $h_3$ . The difference equations (17) are “central” differences.

At boundaries and interfaces it is convenient to use “forward” and “backward” differences. Such difference equations are one-sided in that they express a boundary point in terms of neighboring points interior to the boundary. For the present problem, only first derivatives are of concern.

To derive such difference equations, expand two points, both lying on one side of the reference point  $I, J$ , by using equation (15) in conjunction with Figure 2. For example, a forward expansion yields

$$\begin{aligned} f(I+1, J) &= f(I, J) + h_2 f_{,y}(I, J) + \frac{1}{2} h_2^2 f_{,yy}(I, J) + O(h_2^3) \\ f(I+2, J) &= f(I, J) + 2h_2 f_{,y}(I, J) + \frac{1}{2} (4h_2^2) f_{,yy}(I, J) + O(h_2^3) \end{aligned} \quad . \quad (18)$$

Subtracting one expression from the other to eliminate the second derivative leads to the difference equation for the first derivative. Thus, the forward differences are

$$\begin{aligned} f_{,y}(I, J) &= \frac{1}{2h_2} \left[ 4f(I+1, J) - 3f(I, J) - f(I+2, J) \right] - O(h_2^2) \\ f_{,z}(I, J) &= \frac{1}{2h_3} \left[ 4f(I, J+1) - 3f(I, J) - f(I, J+2) \right] - O(h_3^2) \end{aligned} \quad . \quad (19)$$

Similarly, the backward differences are

$$\begin{aligned} f_{,y}(I, J) &= \frac{1}{2h_1} \left[ 3f(I, J) + f(I-2, J) - 4f(I-1, J) \right] + O(h_1^2) \\ f_{,z}(I, J) &= \frac{1}{2h_3} \left[ 3f(I, J) + f(I, J-2) - 4f(I, J-1) \right] + O(h_3^2) \end{aligned} \quad . \quad (20)$$

It should be pointed out that more simplified, but less accurate, forward and backward expressions can be written; however, the present application requires all the accuracy that it is possible to attain near the free boundaries. Thus, the higher order difference was chosen. In addition, this choice yields a magnitude of error equal to that found in equations (17).

Using the representations just obtained, equations (11) through (14) can be transformed into difference equations characterizing the problem. For example, the last equation in (11) becomes

$$\begin{aligned}
 & \frac{h_1 h_2}{2h_3(h_1 + h_2)} \left\{ (c_{36}^m + c_{45}^m) [U(I+1, J+1) - U(I-1, J+1) - U(I+1, J-1) \right. \\
 & \quad + U(I-1, J-1)] + (c_{23}^m + c_{44}^m) [V(I+1, J+1) \\
 & \quad - V(I-1, J+1) - V(I+1, J-1) + V(I-1, J-1)] \left. \right\} \\
 & + \frac{2h_1}{h_1 + h_2} c_{44}^m [W(I+1, J) + \frac{h_2}{h_1} W(I-1, J)] \\
 & + \frac{h_1 h_2}{h_3^2} c_{33}^m [W(I, J+1) + W(I, J-1)] \\
 & - 2(c_{44}^m + \frac{h_1 h_2}{h_3^2} c_{33}^m) W(I, J) = -h_1 h_2 [c_{13}^m C_2 + c_{23}^m D_V \\
 & + 2c_{36}^m C_4] \quad , \tag{21}
 \end{aligned}$$

where the layer number,  $m$ , is left off  $U$ ,  $V$ , and  $W$  since their location is determined by the node  $I, J$ .

### Developing the Matrix Equation

In this section, the difference equations, like (21), are transformed into a linear matrix equation of the form

$$[A] [X] = [B] \quad , \tag{22}$$

where  $A$  is an  $N \times N$  coefficient matrix ( $N$  being the number of unknowns or equations),  $X$  is the solution vector, and  $B$  is the load or input vector. To accomplish this, the three unknowns ( $U$ ,  $V$ , and  $W$ ) must be uniquely collapsed into the single unknown  $X$  so that at each node three unique equations in  $X$  will be created. For instance, let

$$\begin{array}{l}
 \left. \begin{array}{l} U \rightarrow X(1) \\ V \rightarrow X(2) \\ W \rightarrow X(3) \end{array} \right\} \quad \text{at Node 1} \\
 \left. \begin{array}{l} U \rightarrow X(4) \\ V \rightarrow X(5) \\ W \rightarrow X(6) \end{array} \right\} \quad \text{at Node 2} \quad . \quad (23)
 \end{array}$$

It remains to generalize such a transformation for all nodes.

It is convenient to follow Pipes [1] and his notation is adopted. If LAT is the number of nodes in one column along the vertical axis (LAminate Thickness direction), then the nodes, unknowns, and equations can be identified by a unique number in terms of the nodal position (I, J). If

$$JJ1 = 3[LAT(I - 1) + J] - 2 , \quad (24)$$

then

$$\begin{aligned}
 \text{NODE} &= LAT(I - 1) + J \\
 U(I, J) &= X(JJ1) \\
 V(I, J) &= X(JJ1 + 1) \\
 W(I, J) &= X(JJ1 + 2)
 \end{aligned} \quad (25)$$

and

$$\begin{aligned}
 \text{Number the 1st equation: } &JJ1 \\
 \text{Number the 2nd equation: } &JJ1 + 1 \\
 \text{Number the 3rd equation: } &JJ1 + 2 \quad . \quad (26)
 \end{aligned}$$

Letting I = 1 and J = 1, 2 consecutively generates the results in (23).

Since the finite-difference equations involve unknowns at nodes neighboring the JJ1 node, it is necessary to develop transformation relations like (24) in order to number unknowns at these nodes as well. For example, using I, J as the reference node, a

transformation relation for an unknown at the node  $I - 1, J + 1$  is found by letting  $I \rightarrow I - 1$  and  $J \rightarrow J + 1$  in (24) and giving the result a unique name, for example JJ7. Thus,

$$JJ7 = 3[LAT(I - 2) + J] + 1 \quad . \quad (27)$$

Using Table 1, which identifies all the unknowns at nodes neighboring  $I, J$ , and following the above procedure yields the transformation relations that uniquely number each unknown. In summary, all of these transformations are

$$JJ1 = 3*(LAT*I1 + J) - 2$$

$$JJ2 = 3*(LAT*I2 + J) - 2$$

$$JJ3 = 3*(LAT*I2 + J) - 5$$

$$JJ4 = 3*(LAT*I + J) - 2$$

$$JJ5 = 3*(LAT*I + J) + 1$$

$$JJ6 = 3*(LAT*I1 + J) + 1$$

$$JJ7 = 3*(LAT*I2 + J) + 1$$

$$JJ8 = 3*(LAT*I1 + J) - 5$$

$$JJ9 = 3*(LAT*I + J) - 5$$

$$JJ10 = 3*(LAT*I1 + J) - 8$$

$$JJ11 = 3*(LAT*(I + 1) + J) - 2$$

$$JJ12 = 3*(LAT*I1 + J) + 4$$

$$JJ13 = 3*(LAT*(I - 3) + J) - 2 \quad , \quad (28)$$

where

$$I1 = I - 1$$

$$(29)$$

$$I2 = I - 2$$

TABLE 1. NODE IDENTIFICATION

Node	U	V	W
I, J	X(JJ1)	X(JJ1 + 1)	X(JJ1 + 2)
I - 1, J	X(JJ2)	X(JJ2 + 1)	X(JJ2 + 2)
I - 1, J - 1	X(JJ3)	X(JJ3 + 1)	X(JJ3 + 2)
I + 1, J	X(JJ4)	X(JJ4 + 1)	X(JJ4 + 2)
I + 1, J + 1	X(JJ5)	X(JJ5 + 1)	X(JJ5 + 2)
I, J + 1	X(JJ6)	X(JJ6 + 1)	X(JJ6 + 2)
I - 1, J + 1	X(JJ7)	X(JJ7 + 1)	X(JJ7 + 2)
I, J - 1	X(JJ8)	X(JJ8 + 1)	X(JJ8 + 2)
I + 1, J - 1	X(JJ9)	X(JJ9 + 1)	X(JJ9 + 2)
I, J - 2	X(JJ10)	X(JJ10 + 1)	X(JJ10 + 2)
I + 2, J	X(JJ11)	X(JJ11 + 1)	X(JJ11 + 2)
I, J + 2	X(JJ12)	X(JJ12 + 1)	X(JJ12 + 2)
I - 2, J	X(JJ13)	X(JJ13 + 1)	X(JJ13 + 2)

Generation of the matrix equation (22) now remains. To do this, straightforward substitution for U, V, and W, using Table 1, into equations (11) through (14) yields the desired results in equation form. For example, equation (21) becomes

$$\begin{aligned}
& \frac{h_1 h_2}{2h_3(h_1 + h_2)} \left\{ (c_{36}^m + c_{45}^m) [X(JJ5) - X(JJ7) - X(JJ9) + X(JJ3)] \right. \\
& + (c_{23}^m + c_{44}^m) [X(JJ5 + 1) - X(JJ7 + 1) - X(JJ9 + 1) \\
& + X(JJ3 + 1)] \left. \right\} + \frac{2h_1}{h_1 + h_2} c_{44}^m [X(JJ4 + 2) + \frac{h_2}{h_1} X(JJ2 + 2)] \\
& + \frac{h_1 h_2}{h_3^2} c_{33}^m [X(JJ6 + 2) + X(JJ8 + 2)] \\
& - 2(c_{44}^m + \frac{h_1 h_2}{h_3^2} c_{33}^m) X(JJ1 + 2) \\
& = -h_1 h_2 [c_{13}^m C_2 + c_{23}^m D_V + 2c_{36}^m C_4] \quad . \quad (30)
\end{aligned}$$

To assure non-zero diagonal terms in the A-matrix, an appropriate equation number for (30) is JQ2 (in this case there is only one possibility) where

$$JQ2 = JJ1 + 2 \quad . \quad (31)$$

Now, from equation (30), the only nonzero elements for the JQ2 row in the A-matrix are

$$\begin{aligned}
A(JQ2, JJ5) &= A(JQ2, JJ3) = C \\
A(JQ2, JJ7) &= A(JQ2, JJ9) = -C \\
A(JQ2, JJ5 + 1) &= A(JQ2, JJ3 + 1) = D \\
A(JQ2, JJ7 + 1) &= A(JQ2, JJ9 + 1) = -D \\
A(JQ2, JJ4 + 2) &= 2h_1 c_{44}^m / (h_1 + h_2) \\
A(JQ2, JJ2 + 2) &= (h_2/h_1) \cdot 2h_1 c_{44}^m / (h_1 + h_2) \\
A(JQ2, JJ6 + 2) &= A(JQ2, JJ8 + 2) = h_1 h_2 c_{33}^m / h_3^2 \\
A(JQ2, JJ1 + 2) &= -2(c_{44}^m + h_1 h_2 c_{33}^m / h_3^2) \quad , \quad (32)
\end{aligned}$$

where

$$\begin{aligned} C &= h_1 h_2 (c_{36}^m + c_{45}^m) / 2h_3(h_1 + h_2) \\ D &= h_1 h_2 (c_{23}^m + c_{44}^m) / 2h_3(h_1 + h_2) \end{aligned} \quad . \quad (33)$$

Note that the material constants  $c_{44}^m$  and  $c_{33}^m$  are non-zero ensuring a non-zero diagonal element  $A(JQ2, JJ1 + 2)$ . In addition to this, the load vector is

$$B(JQ2) = -h_1 h_2 [c_{13}^m C_2 + c_{23}^m D_V + 2c_{36}^m C_4] \quad . \quad (34)$$

Of course, these results only apply to node numbers where the third equilibrium equation in (11) holds. The computer program logically connects appropriate equations with each node. The matrix elements for the remaining equations (11) through (14) are generated in a similar fashion.

### The Mesh Simulation

The continuum is to be simulated by a number of nodal points that form a finite-difference mesh. The mesh is distributed over a cross section of the laminate as shown in Figure 3. The mesh is defined by the following parameters:

NLAY: the number of laminae

LAT: the number of nodes along one column in the LAminate Thickness direction

LAW: the number of nodes along one row in the LAminate Width direction

FSW1: the first change in nodal spacing termed Fine Simulation Width

K: magnification factor of the fine simulation width

H: the fine simulation width

Given these parameters, the following parameters can be determined:

INF(M): values of J at the upper INterFace of the mth layer

FSW2: the second change in nodal spacing

KH: the coarse simulation width (K = 1, 2, 3, ...)

JQMAX = 3\*LAT\*LAW: the number of unknowns or equations

IBW = 2\*(3\*LAT + 1): the half bandwidth

NBAND = 2\*IBW + 1: the full band

The bandwidth of the coefficient matrix is found by considering that the maximum number of nodes involved in the difference equations is three, as can be seen from expressions (19) and (20), and calculating the maximum number of consecutive elements on both sides of the diagonal to and including the last off-diagonal non-zero element.

Selecting equations representing the conditions to be imposed at each node remains to be accomplished. Because of the arbitrariness of the corner conditions, a number of choices are possible. Those selected for this program are illustrated in Figure 4.

A user's guide and a more detailed description of the computer program are presented in Appendix C. A program listing is provided also in Appendix C.

## RESULTS

The results given below were obtained using a square mesh, magnification factor K = 1, of size (LAW, LAT) = (13, 9). A complete mesh description, taken from the program output, is displayed in Table 2. It is seen that these dimensions represent a beam rather than a plate. The program was run on an IBM 370 computer utilizing virtual storage.

A single material having properties typical of a high modulus graphite-epoxy was chosen for the above mesh. Using standard notation,

$$E_{11} = 20.0 \times 10^6 \text{ psi} , \quad \nu_{12} = \nu_{13} = \nu_{23} = 0.21$$

$$E_{22} = E_{33} = 2.1 \times 10^6 \text{ psi}$$

$$G_{12} = G_{13} = G_{23} = 0.85 \times 10^6 \text{ psi} ,$$

TABLE 2. MESH DESCRIPTION TAKEN FROM PROGRAM OUTPUT

\*\*\* UNIFORM BENDING OF A LAMINATED PLATE \*\*\*

\*\*\* INPUT DATA \*\*\*

NUMBER OF LAYERS IN CROSS SECTION, NLAY = 4

NUMBER OF NODES ON VERTICAL AXES, LAT = 13

NUMBER OF NODES ON HORIZONTAL AXES, LAT = 9

CHANGE IN MESH WIDTH (FSWL) AT I = 3

CHANGE IN MESH WIDTH (FSW2) AT I = 7

MESH WIDTH MAGNIFICATION FACTOR, K = 1

LAYER NO. 1 INTERFACE AT J = 4

LAYER NO. 2 INTERFACE AT J = 7

LAYER NO. 3 INTERFACE AT J = 10

LAYER NO. 4 INTERFACE AT J = 13

FINE SIMULATION WIDTH, H = 0.00167

where the subscript "1" refers to the fiber direction. The two laminate configurations which are considered are

$$A = [\theta, 0, 0, \theta]$$

and

$$B = [0, \theta, \theta, 0]$$

with  $\theta$  as in Figure 1 such that 0 degree  $\leq \theta \leq 90$  degrees. Typical laminate data and load constants are displayed in Table 3.<sup>3</sup> Here the additional constant MT is the resulting moment required to produce a specified maximum strain which, for the present analysis, is  $\epsilon_x = 1.0 \times 10^{-3}$  inch/inch (see Appendix B).

A sample of the results for the displacement functions U, V, and W is presented in Table 4. Examination of their variation with respect to z reveals the apparent symmetry relations,

U, V      antisymmetric in z

W      symmetric in z

within an accuracy of two digits.

Symmetries with respect to y are evident for the strains within three-digit accuracy. Samples of these results are plotted in Figures 5 and 6. Coupling these apparent symmetries with the strain relations (2) in an expanded form yields

U, V      antisymmetric in y

W      symmetric in y

The displacement results verify this precisely for U (to four places), but show some deviation in V and W.<sup>4</sup>

To illustrate the effect of bending on the stress distribution, Figures 7 through 19 are presented. Although convergence to the exact values has yet to be demonstrated, the results do have qualitative merit. The following cases result from a bending strain of  $\epsilon_x = 1.0 \times 10^{-3}$  inch/inch prescribed at the bottom surface.

Of principal interest are the interlaminar stresses illustrated in Figures 7 through 12. We note that laminates composed of 30 degree or 45 degree layers produce the greatest stress rise in  $\sigma_z$  at the free edge with a more pronounced effect occurring if the angle plies are on the outside, i.e., system A =  $[\theta, 0, 0, \theta]$ . A similar effect is seen in the shear stress  $\tau_{yz}$ , although the rise in stress is sharply blunted by the requirement of zero

3. The thermal problem is neglected in this preliminary analysis even though expansion coefficients appear in the program.

4. It is interesting to note that the y-symmetries for V and W are verified precisely using the coarser mesh (LAW, LAT) = (8, 9) which decreases the relative size of the bandwidth.

TABLE 3. TYPICAL LAMINATE DATA AND LOAD CONSTANTS TAKEN  
FROM PROGRAM OUTPUT

*** MATERIAL DATA ***									
LAYER	E11	E22	E33	E12	E13	E23	N112	N113	N1123
1	20.000E+06	2.100E+06	0.350E+06	3.850E+06	0.850E+06	0.21	0.21	0.21	0.21
2	20.000E+06	2.100E+06	0.350E+06	0.950E+06	0.850E+06	0.21	0.21	0.21	0.21
3	20.000E+06	2.100E+06	0.350E+06	0.850E+06	0.850E+06	0.21	0.21	0.21	0.21
4	20.000E+06	2.100E+06	0.350E+06	0.350E+06	0.850E+06	0.21	0.21	0.21	0.21
*** STIFFNESS MATRICES ***									
X-Y-Z MATRIX									
1	-6.745E+12	-5.145E+06	5.210E+05	-0.0	-0.0	-4.536E+06	-2.0240E+07	-5.6480E+05	-0.0
							-4.536E+06	-5.6480E+05	-0.0
2	-6.745E+06	-5.210E+05	-0.0	0.0	0.0	-4.536E+06	-2.2130E+06	-4.7710E+05	0.0
							-4.536E+06	-4.7710E+05	0.0
3	-2.213E+06	-0.0	-0.0	-0.0	-0.0	-2.2130E+06	-0.0	-0.0	-0.0
							-2.2130E+06	-0.0	-0.0
4	0.500E+05	-0.0	0.0	0.0	0.0	0.500E+05	0.0	0.0	0.0
							0.500E+05	0.0	0.0
								0.500E+05	0.0
									0.500E+05
X-Y-Z PRIME-MATRIX									
1	-2.024E+12	-5.348E+05	5.648E+05	-0.0	-0.0	-2.0240E+12	-5.6480E+05	-5.6480E+05	-0.0
							-2.0240E+12	-5.6480E+05	-0.0
2	-2.024E+06	-4.771E+05	-0.0	0.0	0.0	-2.2130E+06	-4.7710E+05	-0.0	-0.0
							-2.2130E+06	-4.7710E+05	-0.0
3	-2.213E+06	-0.0	-0.0	-0.0	-0.0	-2.2130E+06	-0.0	-0.0	-0.0
							-2.2130E+06	-0.0	-0.0
4	0.0	-0.500E+05	-0.0	0.0	0.0	0.500E+05	-0.0	-0.0	-0.0
							0.500E+05	-0.0	-0.0
								0.500E+05	-0.0
X-Y-Z PRIME-MATRIX									
1	-2.024E+12	-5.348E+05	5.648E+05	-0.0	-0.0	-2.0240E+12	-5.6480E+05	-5.6480E+05	-0.0
							-2.0240E+12	-5.6480E+05	-0.0
2	-2.024E+06	-4.771E+05	-0.0	0.0	0.0	-2.2130E+06	-4.7710E+05	-0.0	-0.0
							-2.2130E+06	-4.7710E+05	-0.0
3	-2.213E+06	-0.0	-0.0	-0.0	-0.0	-2.2130E+06	-0.0	-0.0	-0.0
							-2.2130E+06	-0.0	-0.0
4	0.0	-0.500E+05	-0.0	0.0	0.0	0.500E+05	-0.0	-0.0	-0.0
							0.500E+05	-0.0	-0.0
								0.500E+05	-0.0

TABLE 3. (Concluded)

4	$6.745E+36$	$5.145E+06$	$5.210E+05$	$0.0$	$3.0$	$4.596E+06$	$2.0240E+37$	$5.6480E+05$	$5.6480E+05$	$0.0$	$0.0$
	$6.745E+06$	$5.210E+05$	$0.0$		$9.0$	$4.506E+06$	$2.02130E+06$	$4.7710E+05$	$0.0$	$0.0$	
	$2.02130E+06$	$0.0$					$2.02130E+06$	$0.0$	$0.0$	$0.0$	
$-4.5 = 0$					$9.0$	$4.307E+04$					
							$8.000E+05$	$-9.0$	$0.0$	$0.0$	
							$8.000E+05$	$-9.0$	$0.0$	$0.0$	
							$8.000E+05$	$-9.0$	$0.0$	$0.0$	
							$8.000E+05$	$-9.0$	$0.0$	$0.0$	
							$5.330E+06$				
							$5.330E+06$				
								$9.000E+05$			
								$9.000E+05$			
									$0.000E+00$		

\*\*\* COEFFICIENTS OF THERMAL EXPANSION \*\*\*

LAYER	THETA	AL1	AL2	AL3	AL5	AL1P	AL2P	AL3P
1	$45.0$	$0.600E-05$	$0.001E-35$	$0.120E-04$	$-0.120E-04$	$0.0$	$0.120E-04$	$0.120E-04$
2	$0.0$	$0.120E-04$	$0.120E-04$	$0.0$	$0.0$	$0.0$	$0.120E-04$	$0.120E-04$
3	$0.0$	$0.120E-04$	$0.120E-04$	$0.0$	$0.0$	$0.0$	$0.120E-04$	$0.120E-04$
4	$-45.0$	$0.600E-05$	$0.390E-05$	$0.120E-04$	$-0.120E-04$	$0.0$	$0.120E-04$	$0.120E-04$

\*\*\* THE LOAD-CONSTANTS \*\*\*

$-C2 = -1.000E-31$	$C3 = 3.0$	$C4 = -1.311E-01$	$C5 = 0.0$	$C6 = 0.0$	$C7 = 0.0$	$C8 = 0.0$	$C9 = 0.0$	$C10 = 0.0$

ERROR CONDITION OF SLEEVES ROUTINE IS  $0.0$  PANK IS  $-35E-3$  DETERMINANT =  $1.00$

NOTE: MT IS THE RESULTING MOMENT REQUIRED TO PRODUCE THE SPECIFIED MAXIMUM STRAIN.

TABLE 4. DISPLACEMENT FUNCTION RESULTS TAKEN FROM  
PROGRAM OUTPUT FOR LAMINATE DESCRIBED IN  
TABLES 2 AND 3

\*\*\* GRID POINT DISPLACEMENT FUNCTIONS \*\*\*

NODE	U-DISPLACEMENT	V-DISPLACEMENT	W-DISPLACEMENT
1	0.161561D-04	0.264686D-04	-0.909032D-05
2	0.149580D-04	0.219831D-04	-0.936804D-05
3	0.125381D-04	0.173176D-04	-0.951939D-05
4	0.953594D-05	0.127014D-04	-0.953590D-05
5	0.611696D-05	0.818997D-05	-0.940002D-05
6	0.304395D-05	0.403364D-05	-0.924686D-05
7	0.487189D-09	0.250769D-08	-0.916589D-05
8	-0.304291D-05	-0.403918D-05	-0.925084D-05
9	-0.611575D-05	-0.319432D-05	-0.937698D-05
10	-0.926689D-05	-0.126308D-04	-0.952892D-05
11	-0.125354D-04	-0.173211D-04	-0.954418D-05
12	-0.149550D-04	-0.219880D-04	-0.937161D-05
13	-0.161503D-04	-0.264808D-04	-0.909891D-05

stress at the free edge, and here the stress in system B = [0,  $\theta$ ,  $\theta$ , 0] is slightly more pronounced than that in A. The largest stress rise, an order of magnitude greater than  $\sigma_z$  and  $\tau_{yz}$ , is created in the A-system in  $\tau_{xz}$ . Again it is the 30 degree laminate incurring the sharpest stress rise, but here the 15 degree laminate overshadows the 45 degree laminate. In summary, the laminates containing 15 degree through 45 degree layers located adjacent to 0 degree layers have the largest interlaminar stresses for the cases considered; i.e.,  $0 \text{ degree} \leq \theta \leq 90 \text{ degrees}$  taken in 15 degree intervals.

Some results peculiar to the numerical method of solution should be pointed out. Referring to Figure 9, we note a sharp rise in the stress  $\sigma_z$  at the midpoint node (I, J) = (5, 7). This is a result of fixing the displacements at I = 5 and 6, J = 7 in the program in order to zero-out rigid body motion and drift in the solution routine. However averaging the values for  $\sigma_z$  just above and just below the interface (at J = 7, m = 2 and m = 3) yields a more plausible result. Since the tractions must be continuous at the interface anyway, this averaging technique was also applied at the free edges where the free surface conditions were adopted in lieu of the continuity conditions. This technique had varying success as illustrated by the 75 degree and 90 degree configurations in Figures 10 and 11.

The in-plane stresses are illustrated in Figures 13 through 19. In Figure 13, we find that  $\sigma_x$  in the 0 degree layers is independent of the orientation of the adjacent layer when the maximum strain is specified.<sup>5</sup> This facilitates the presentation of both systems A and B in one figure. It is interesting to note in Figure 14 that  $\sigma_x$  rises at the free edge if the 0 degree layers are outside the laminate and drops if these layers are inside the laminate.

Observation of Figures 15 and 17 for the distribution of  $\sigma_y$  and  $\tau_{xy}$  with respect to z reveals that the off-axis layers, particularly again for 15 degrees through 45 degrees, serve as stress raisers with the effect considerably more pronounced if the 0 degree layers are inside.

Typical distributions of  $\sigma_y$  and  $\tau_{xy}$  with respect to y are shown in Figures 18 and 19. The disturbing feature of these plots is that the stresses just above an interface do not approach zero at the free surface. One cause of this problem is the placement of nodes directly on the interface, which requires their occupation by both layers. Then at the corners, as stated previously, the multitude of boundary conditions cannot be satisfied.<sup>6</sup> However this problem is confined to the free surface nodes and one line of

---

5. In agreement with the beam theory approximation.

6. Placing the interface between two nodal lines may alleviate this problem.

interior nodes. To see this, one may examine the curves for the A-system at  $J = 4-$  and  $J = 10+$  and note that they are reflections of each other within the range  $3 \leq I \leq 7$ . Since, from above,  $\sigma_y$  and  $\tau_{xy}$  appear, in general, to be antisymmetric in  $z$ , the correct values at  $J = 10+$  are recovered within this range if we accept the values at  $J = 4-$ .

## CONCLUSIONS

Although only two types of laminate systems were considered, namely  $A = [\theta, 0, 0, \theta]$  and  $B = [0, \theta, \theta, 0]$ , it is reasonable to assume from these results and from physical considerations that the following symmetry relations hold for balanced ( $B_{ij} = 0$ ) composites:

$U, V$       antisymmetric in  $y$  and  $z$

$W$       symmetric in  $y$  and  $z$

where  $U$ ,  $V$ , and  $W$  are displacement functions of  $y$  and  $z$ . Based on the stress results, laminates containing layers oriented within the range  $15 \text{ degrees} \leq \theta \leq 45 \text{ degrees}$  produce the largest interlaminar stresses out of the cases studied,  $0 \text{ degrees} \leq \theta \leq 90 \text{ degrees}$  taken in 15 degree intervals. In fact this same group of laminates produces high values in the in-plane stresses as well, with the effect considerably more pronounced for the A-system. Although some deviations in stress occur in the numerical solution, they are localized to a double line of nodes at the boundary. This is a disconcerting feature of the solution in that the boundary region stresses appear to be critically involved in delamination-type failure, which makes their accurate determination desirable.

This study provides a base for future work in this area. Using the present program coupled with an out-of-core equation solver routine, unbalanced laminates may be studied. Using the symmetry relations discussed above, the present computer program may be modified to more efficiently handle balanced laminates ( $B_{ij} = 0$ ).

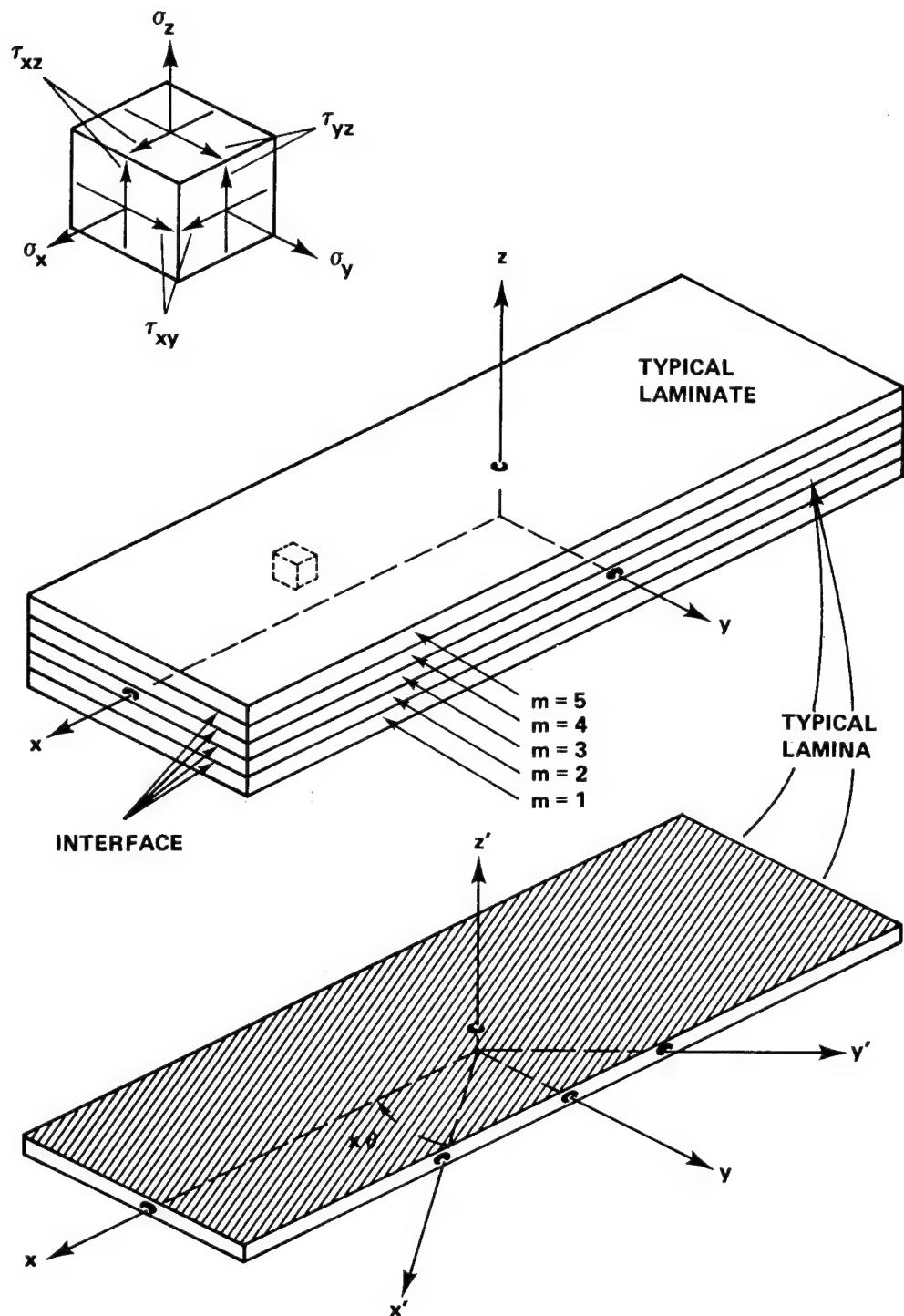


Figure 1. Laminate geometry.

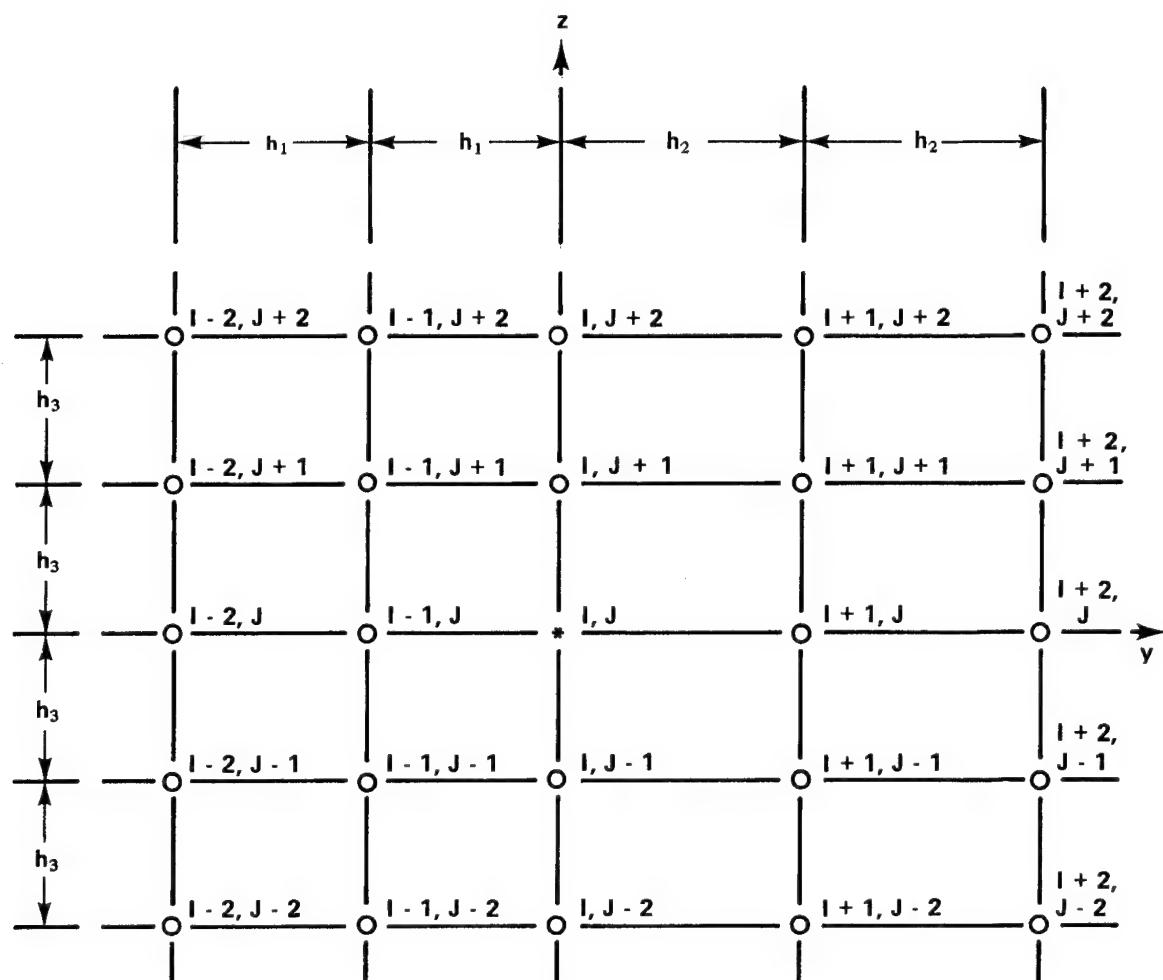


Figure 2. Finite-difference mesh.

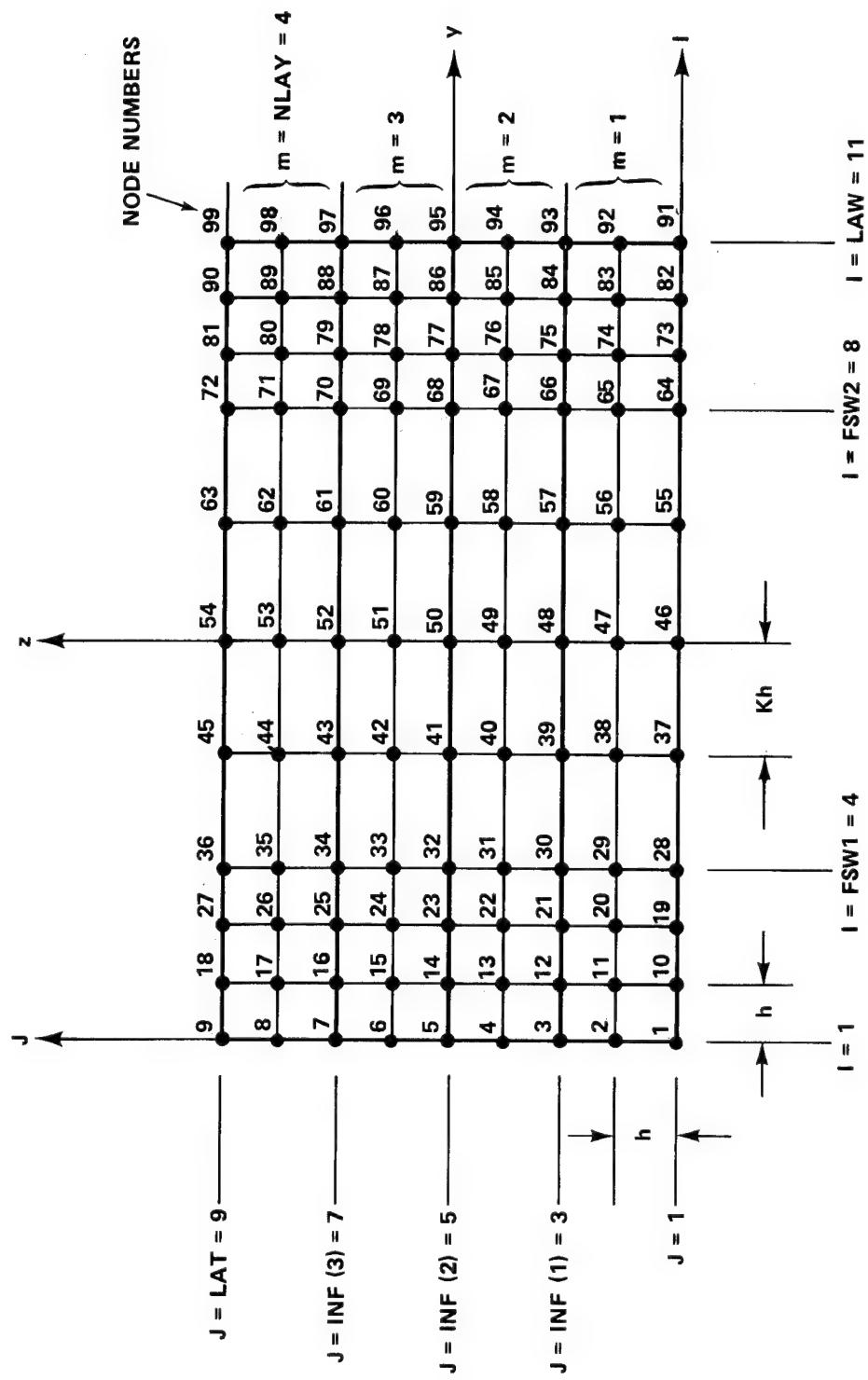
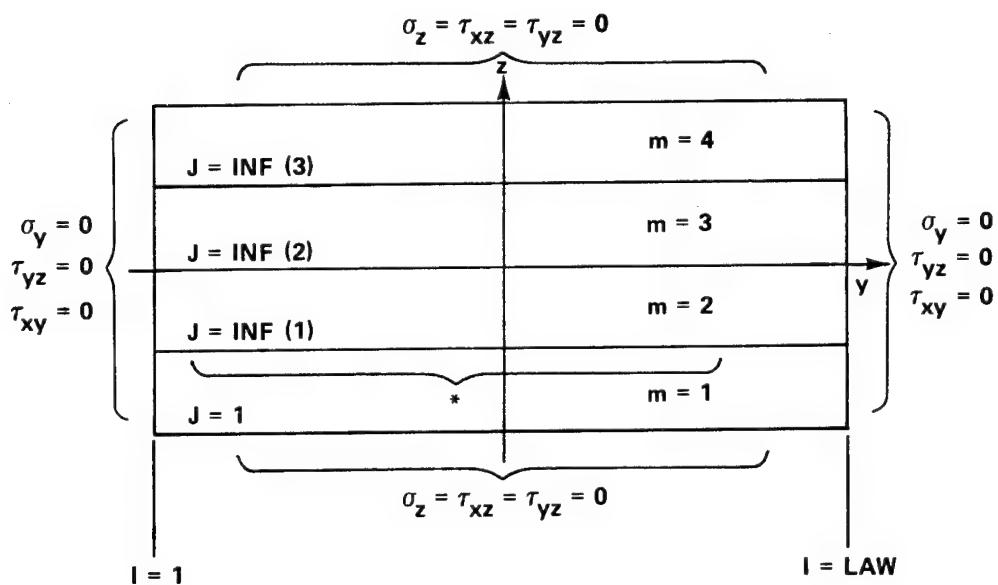
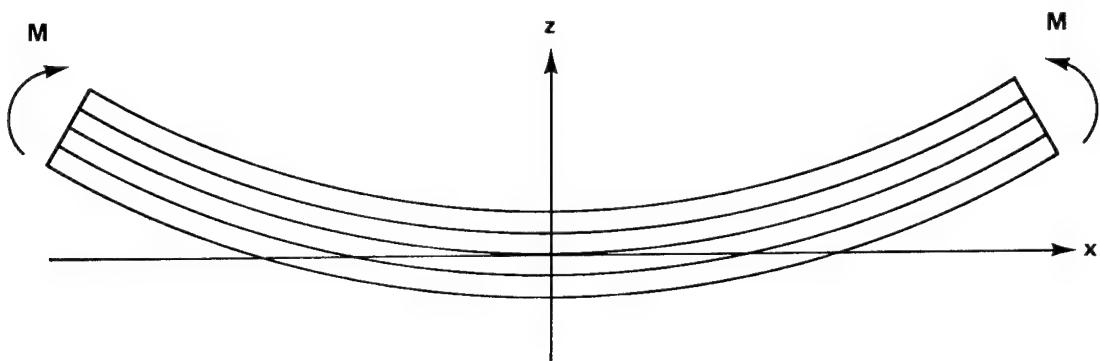


Figure 3. A typical laminate mesh.



\*AT  $\text{INF}(m)$  WHERE  $1 < i < \text{LAW}$  AND  $1 \leq m < \text{NLAY}$ :

$$[u^m, v^m, w^m] = [u^{m+1}, v^{m+1}, w^{m+1}]$$

$$[\sigma_z^m, \tau_{yz}^m, \tau_{xz}^m] = [\sigma_z^{m+1}, \tau_{yz}^{m+1}, \tau_{xz}^{m+1}]$$

- STATIC EQUILIBRIUM IS IMPOSED AT ALL INTERIOR POINTS

Figure 4. Equations selected for each node.

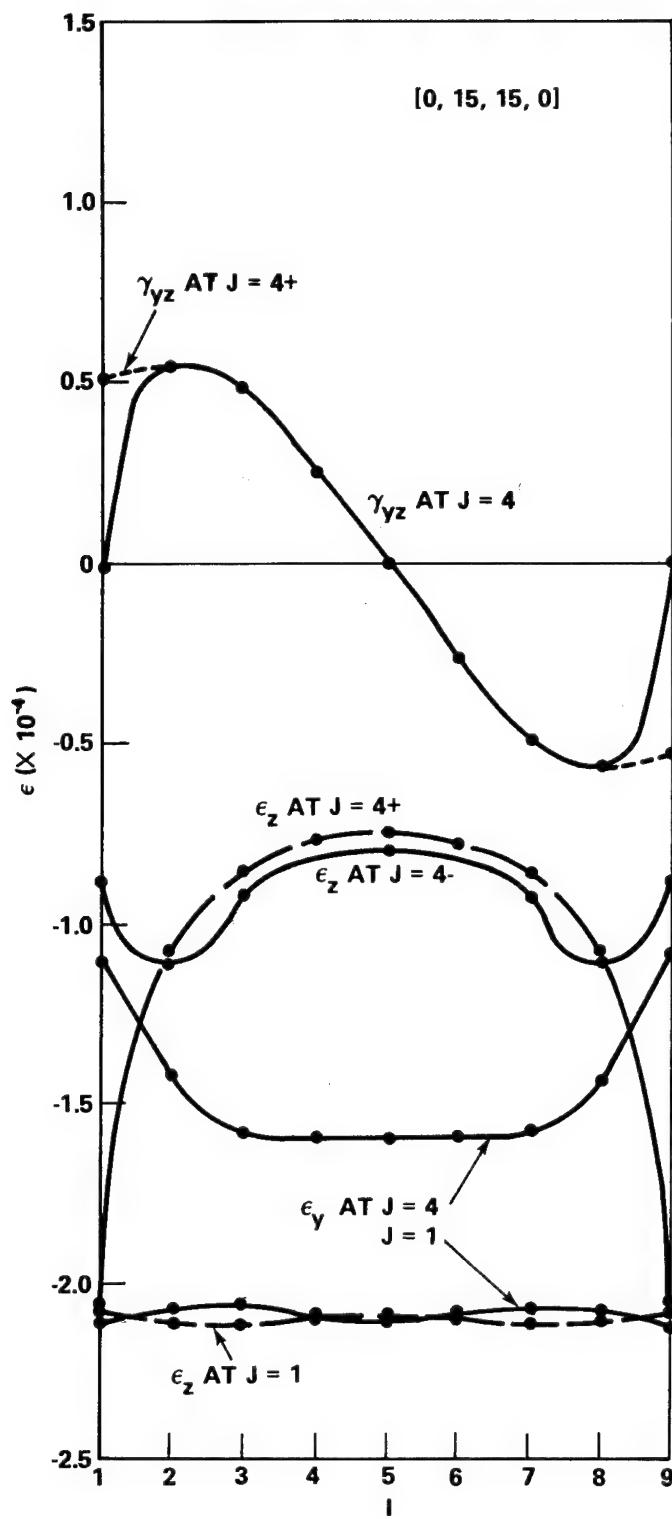


Figure 5. Variation of strain with  $y$ .

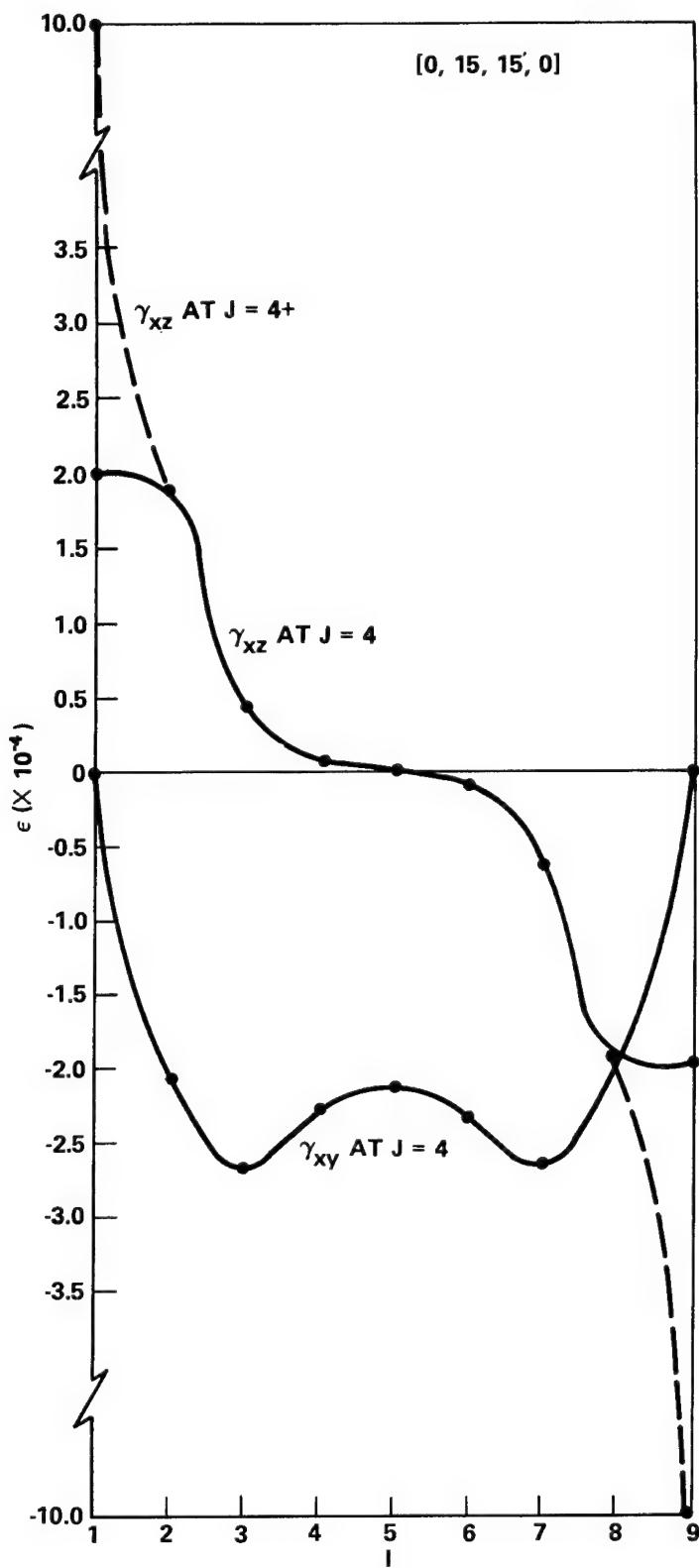


Figure 6. Variation of shear strain with y.

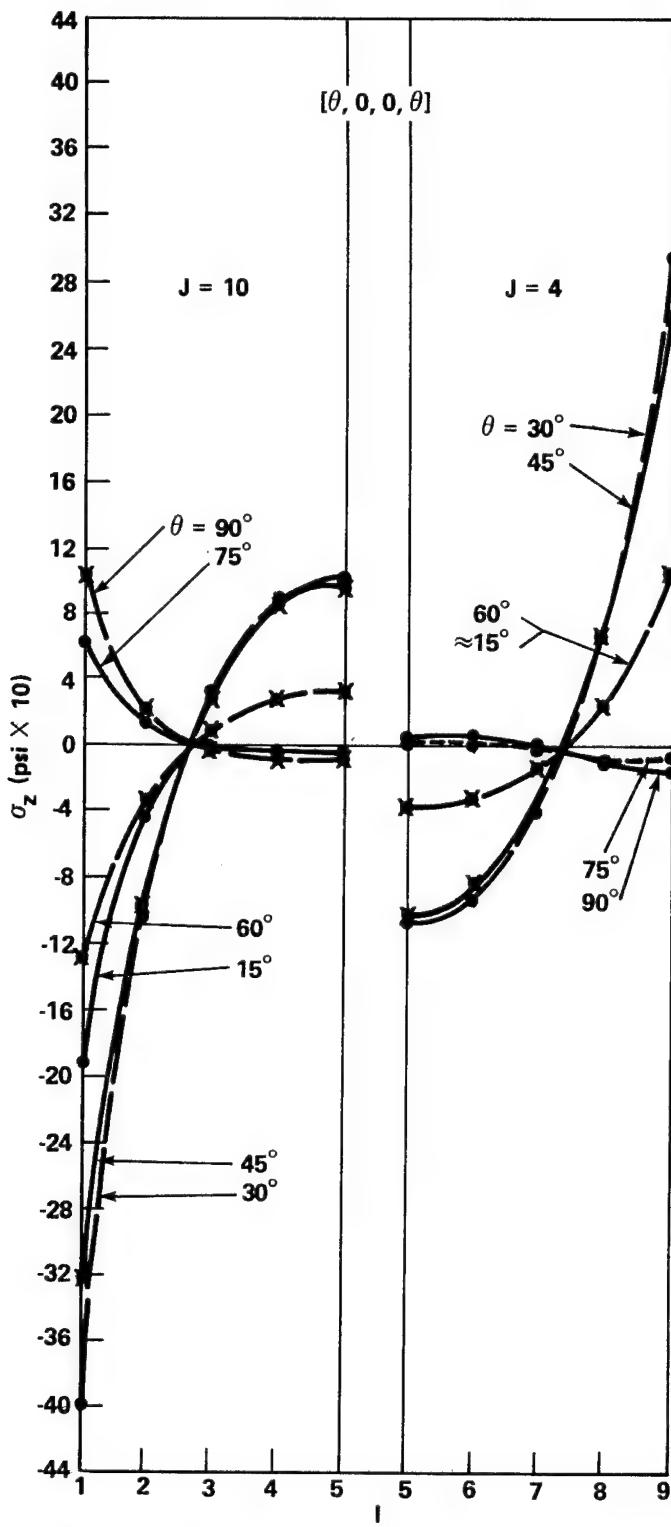


Figure 7. Variation of the normal stress  $\sigma_z$  (symmetric in  $y$ ) with  $y$  for a  $[\theta, 0, 0, \theta]$  laminate.

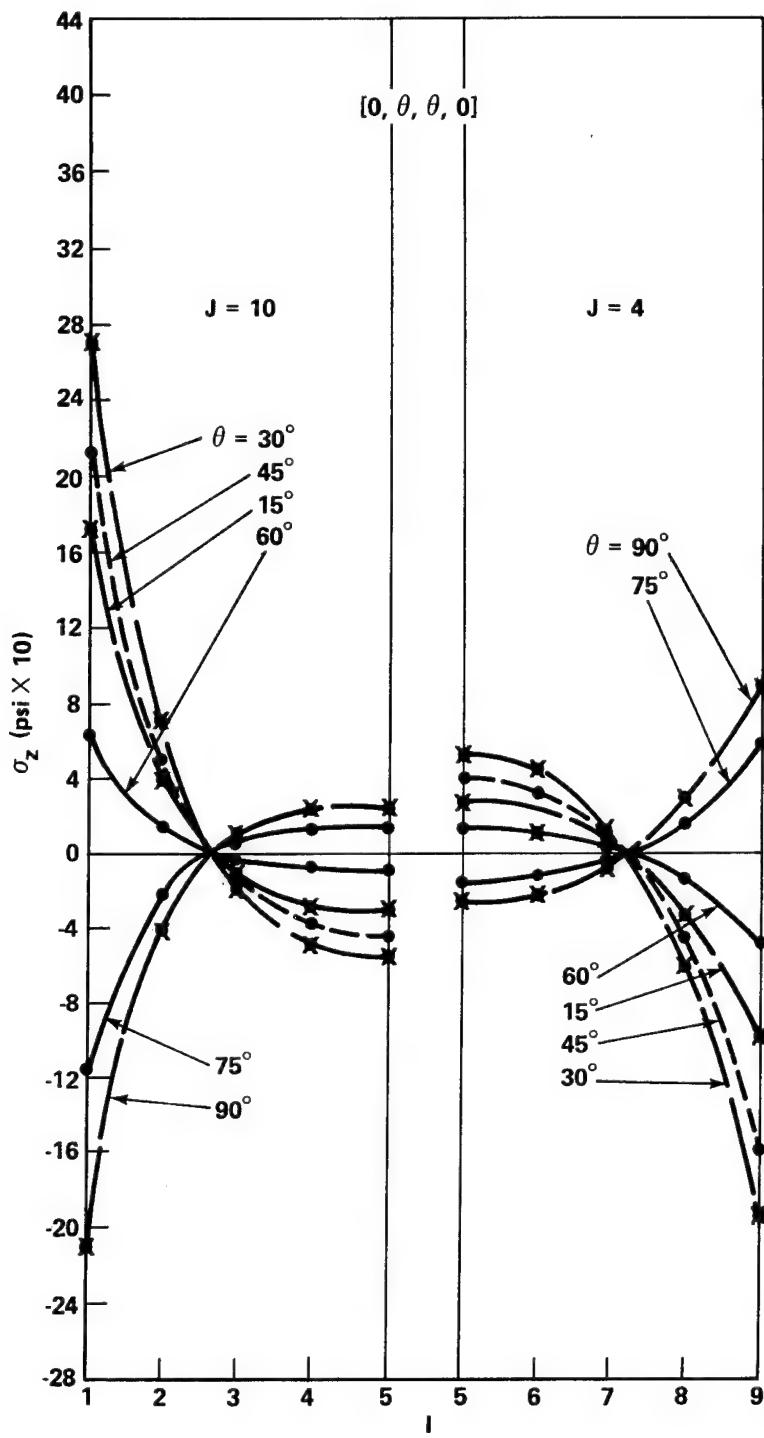


Figure 8. Variation of the normal stress  $\sigma_z$  (symmetric in  $y$ ) with  $y$  for a  $[0, \theta, \theta, 0]$  laminate.

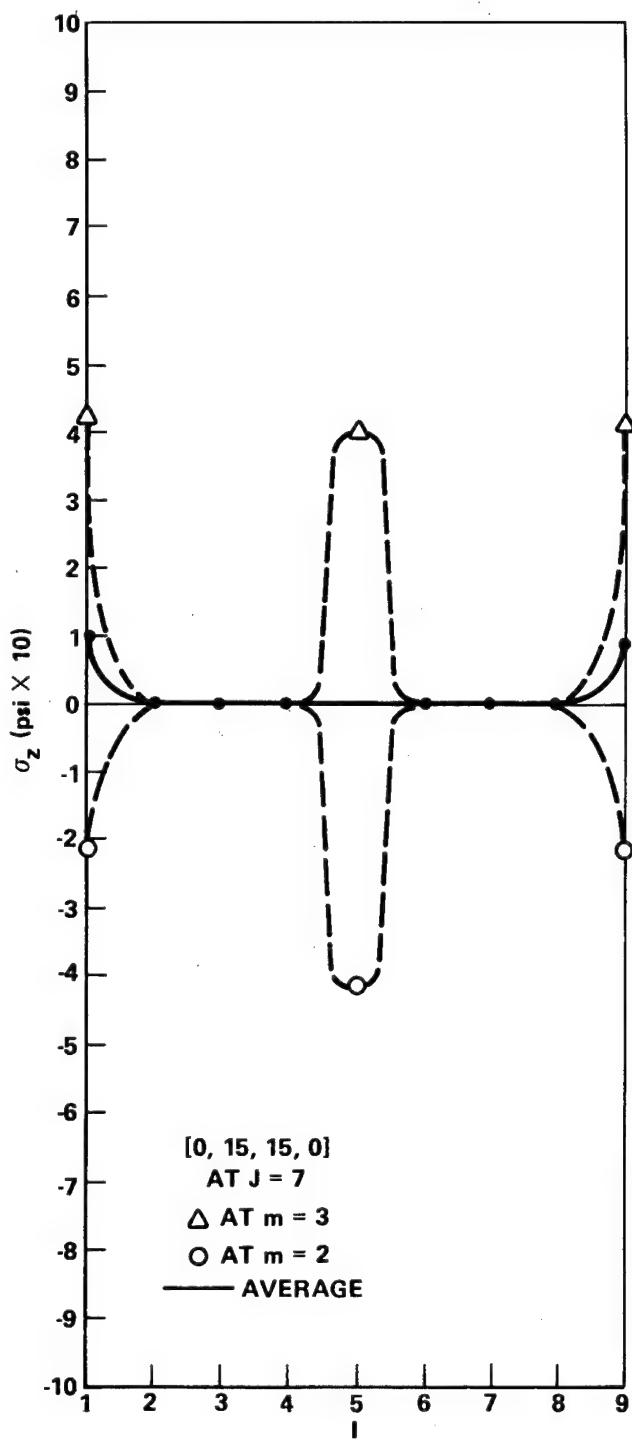


Figure 9. Numerical peculiarities in the normal stress  $\sigma_z$ .

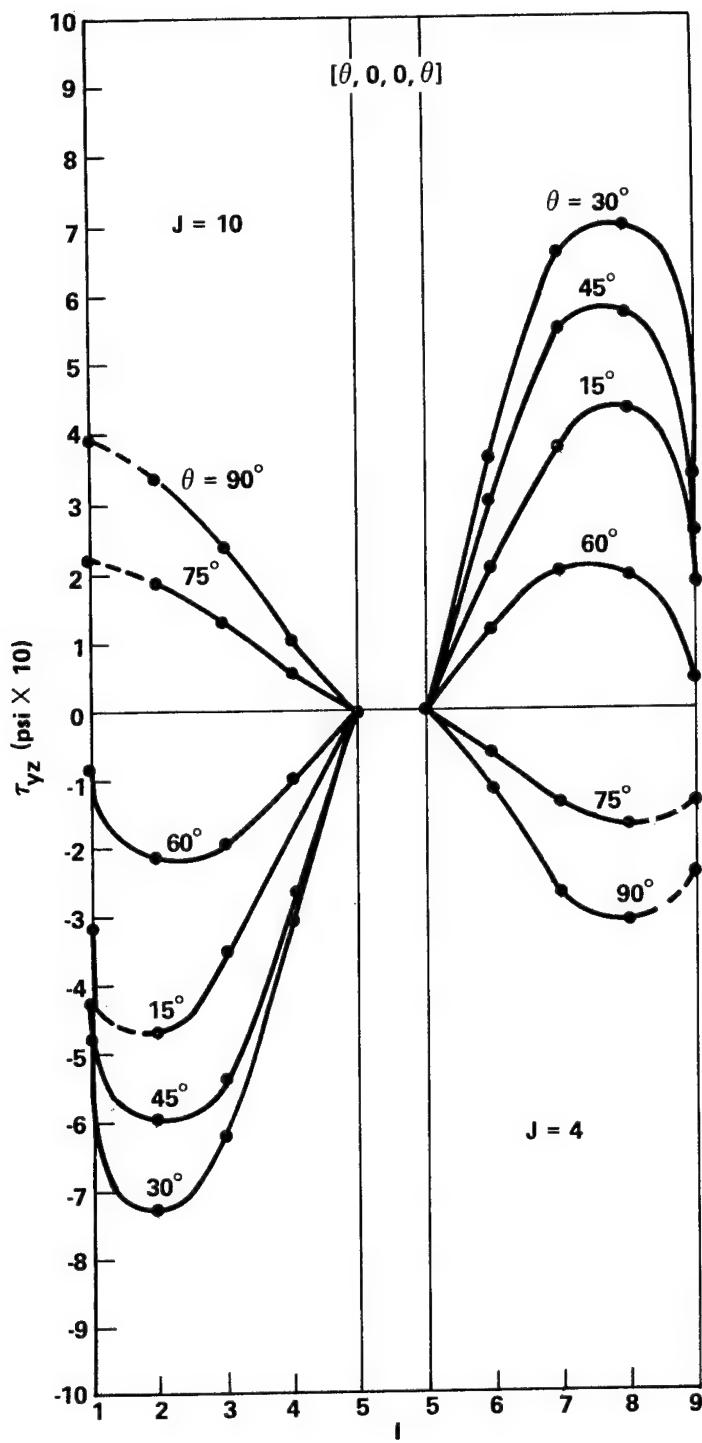


Figure 10. Variation of the shear stress  $\tau_{yz}$  (antisymmetric in  $y$ ) with  $y$  for a  $[\theta, 0, 0, \theta]$  laminate.

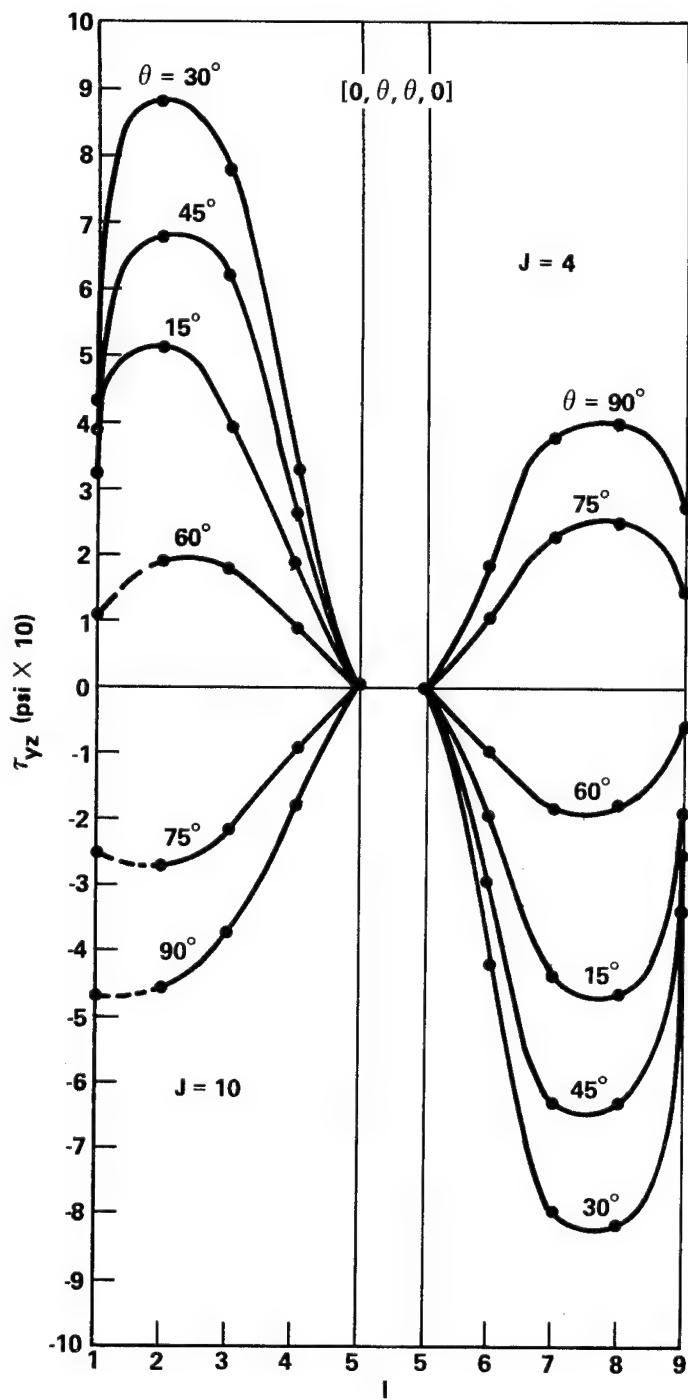


Figure 11. Variation of the shear stress  $\tau_{yz}$  (antisymmetric in  $y$ ) with  $y$  for a  $[0, \theta, \theta, 0]$  laminate.

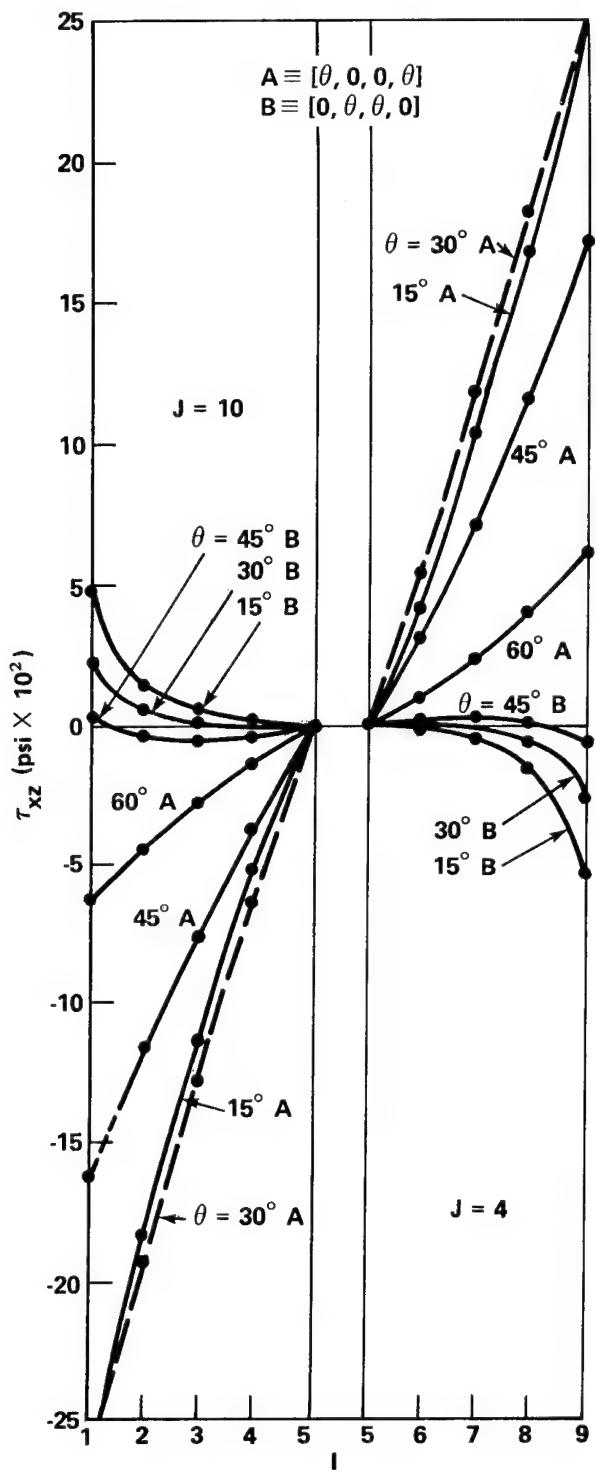


Figure 12. Variation of the shear stress  $\tau_{xz}$  (antisymmetric in  $y$ ) with  $y$ .

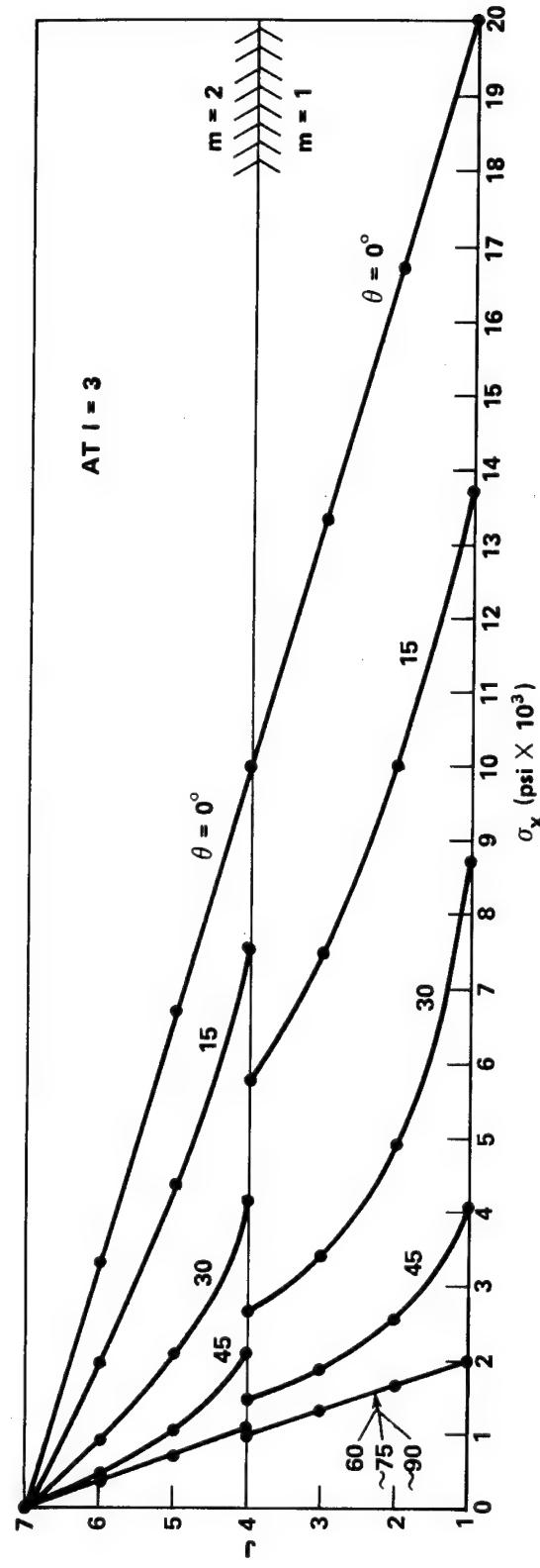


Figure 13. Variation of the normal stress  $\sigma_x$  (antisymmetric in  $z$ ) with  $z$  for each layer with respect to position where the adjacent layer is oriented at  $\theta = 0$  degree.

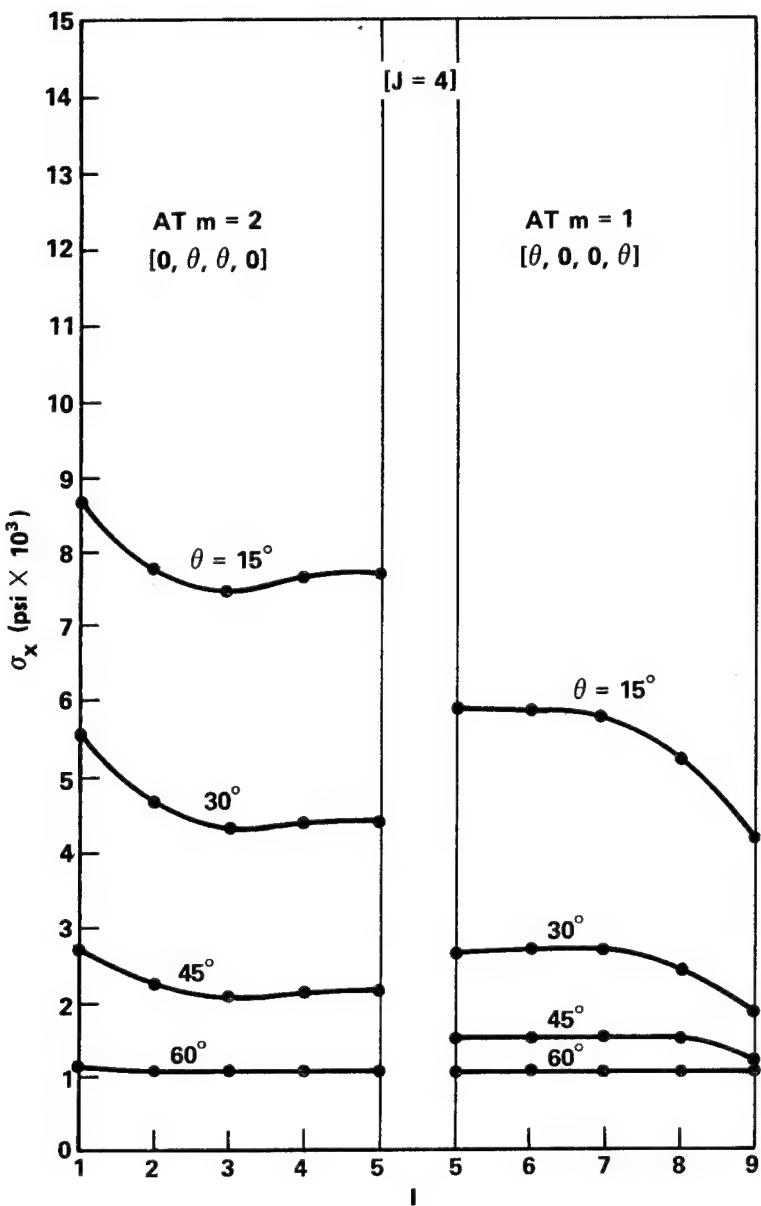


Figure 14. Variation of the normal stress  $\sigma_x$  (symmetric in  $y$ ) with  $y$ .

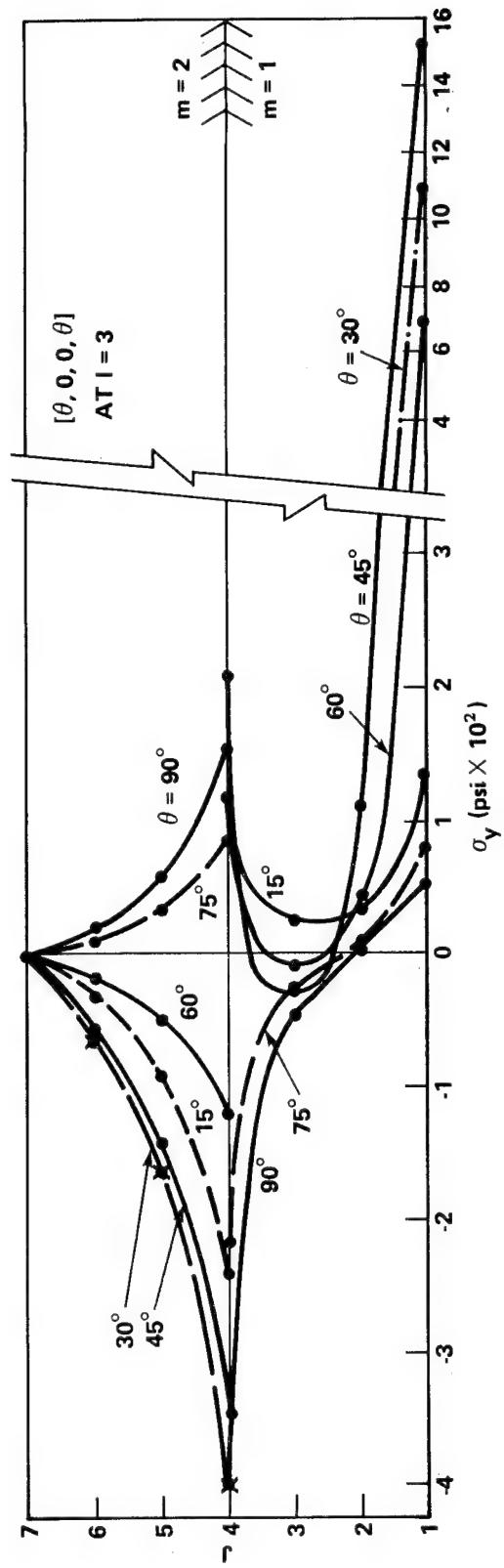


Figure 15. Variation of the normal stress  $\sigma_y$  (antisymmetric in  $z$ ) with  $z$  for a  $[\theta, 0, 0, \theta]$  laminate.

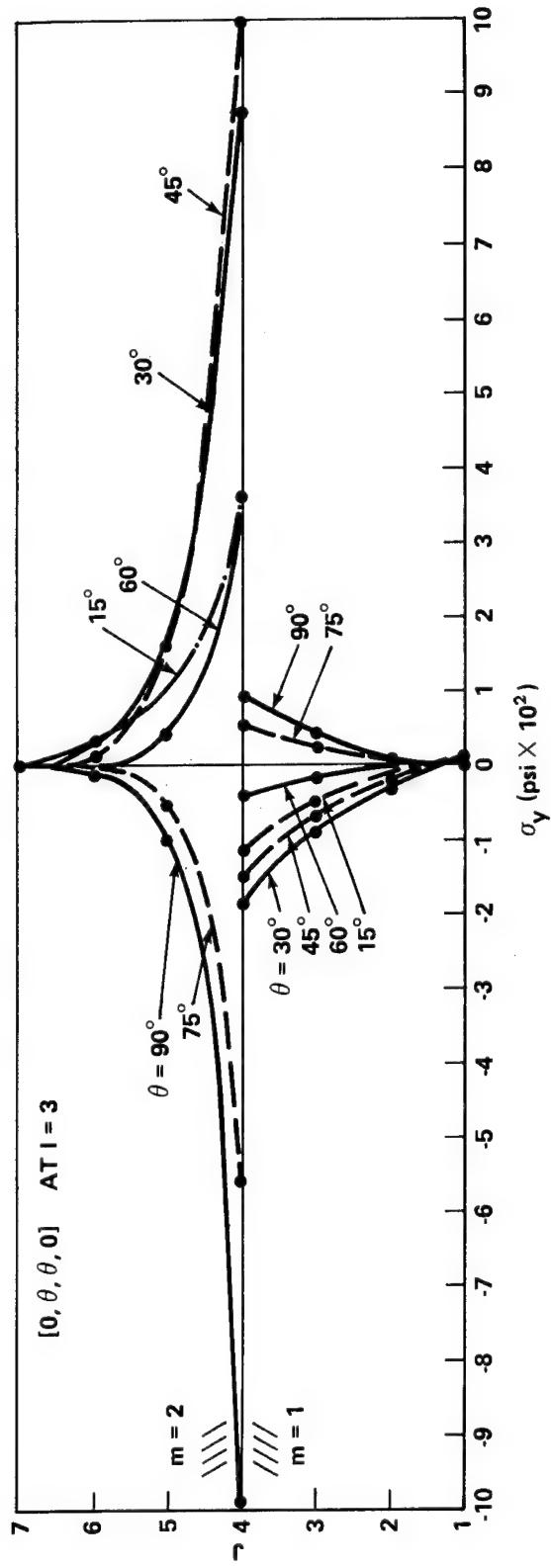


Figure 16. Variation of the normal stress  $\sigma_y$  (antisymmetric in  $z$ ) with  $z$  for a  $[0, \theta, \theta, 0]$  laminate.

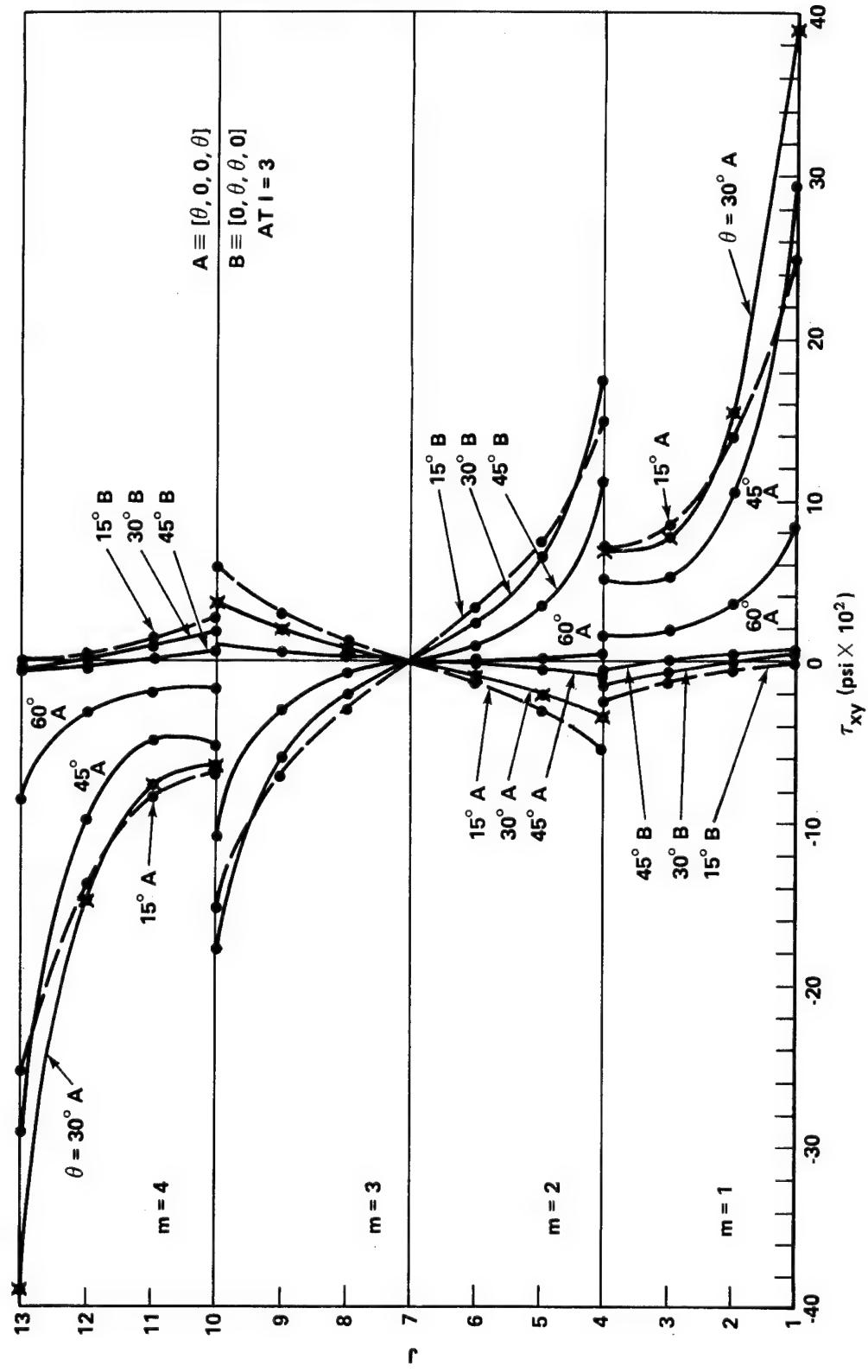


Figure 17. Variation of the shear stress  $\tau_{xy}$  with  $z$ .

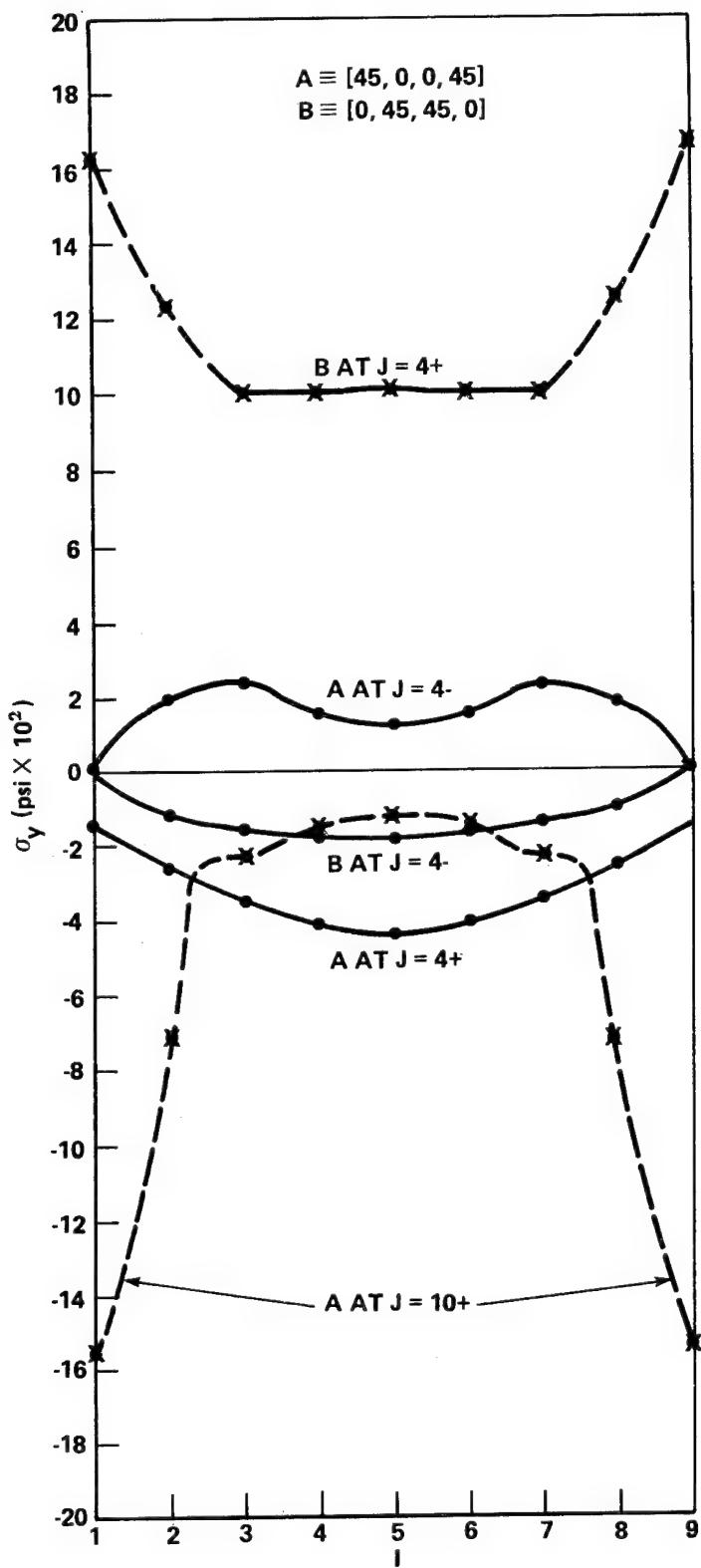


Figure 18. Variation of the normal stress  $\sigma_y$  with  $y$ .

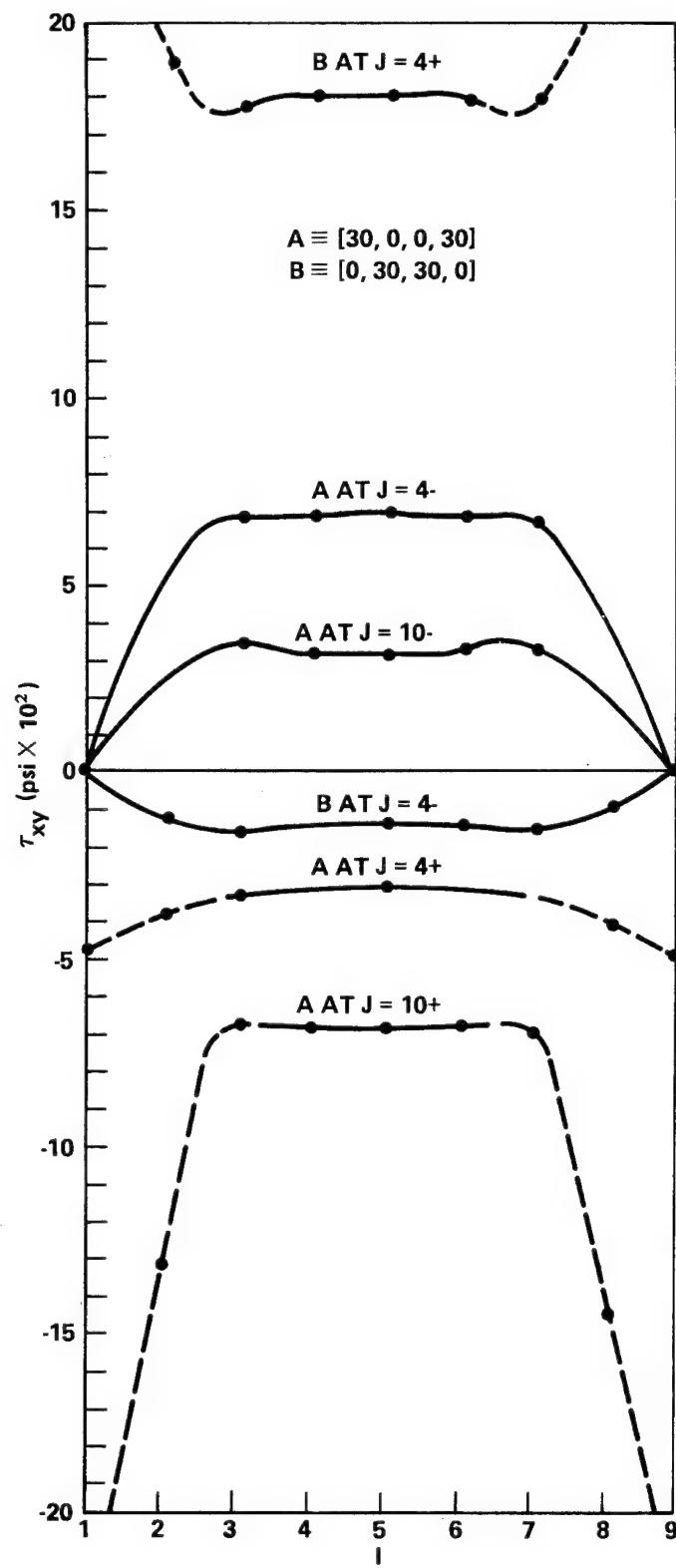


Figure 19. Variation of the shear stress  $\tau_{xy}$  with  $y$ .

## APPENDIX A

### LAMINATE CONSTANTS

Following Reference 9 or 10, define

$$Q_{ij}^m = c_{ij}^m - \frac{c_{i3}^m c_{j3}^m}{c_{33}^m} \quad ; \quad i, j = 1, 2, 6$$

and let  $t$  be the half-thickness of the laminate,  $h_0$  the thickness of a lamina, and  $N$  the total number of layers; then

$$A_{ij} = h_0 \sum_{m=1}^N Q_{ij}^m$$

$$B_{ij} = \frac{h_0^2}{2} \left\{ \sum_{m=1}^N Q_{ij}^m (2m - 1) - N \sum_{m=1}^N Q_{ij}^m \right\}$$

$$D_{ij} = \frac{h_0^3}{3} \left\{ \sum_{m=1}^N Q_{ij}^m (3m^2 - 3m + 1) \right.$$

$$- \frac{3}{2} N \sum_{m=1}^N Q_{ij}^m (2m - 1)$$

$$+ \frac{3}{4} N^2 \sum_{m=1}^N Q_{ij}^m \left. \right\}$$

with  $i, j = 1, 2, 6$ . Finally, let

$$A^* = A^{-1}, \quad B^* = -A^{-1}B, \quad \text{and} \quad D^* = D - BA^{-1}B$$

where the letters symbolize  $3 \times 3$  matrices. Then,

$$B' = B^*(D^*)^{-1}$$

and

$$D' = (D^*)^{-1}$$

Considerable simplification is attained if the laminate is balanced, which implies  $B_{ij} = B'_{ij} = 0$ .

## APPENDIX B

### STRAIN SPECIFICATION

Rather than prescribe the laminate loading as end moments, the maximum strain,  $\epsilon_x^{\max}$ , at the top and bottom surfaces,  $z = \pm z^{\max}$ , will be prescribed. From equation (9), we have

$$\epsilon_x^{\max} = C_2 z^{\max} + C_3$$

Now from equations (5),

$$C_3 = -B'_{11}M = \frac{B'_{11}}{D'_{11}} C_2$$

so that

$$\epsilon_x^{\max} = C_2 \left( z^{\max} + \frac{B'_{11}}{D'_{11}} \right)$$

and, thus,

$$C_2 = \frac{D'_{11} \epsilon_x^{\max}}{B'_{11} + D'_{11} z^{\max}}$$

In the computer program, we set  $\epsilon_x^{\max} = -1.0 \times 10^{-3}$  inch/inch at the top surface  $z = +z^{\max}$  to evaluate the constant  $C_2$  which represents the inverse bending radius.

## APPENDIX C

### THE COMPUTER PROGRAM

#### Program Description

The computer program is an in-core program and is not overlayed. It is felt that a flow chart of the program would be no less complicated than the presentation of a listing with an accompanying explanation, so the latter choice will be followed. Certain statements in the program are extraneous to the problem in this report because the program is in steady transition to handle more general problems. A part-by-part description follows.

Part I. Part I contains a brief definition of terms and an explanation of the order and format of the data cards. The dimensions of the data are: H is in inches, E is material constants in psi (the shear moduli  $G_{12}$ , etc., are read into the E12, etc., arrays), ALPHA is the coefficient of expansion in inches/inch/ $^{\circ}$ F, and THETA is the lamina orientation in angular degrees. Precision and dimension statements are then established, data are read in, and mesh parameters are calculated. The letter M refers to the layer number. In the loop, D0 9000, IRAN counts each laminate layup from one to IRUN (only changes in lamina orientation are allowed for within this loop).

Part II. Part II calculates the anisotropic stiffness matrix. BETA is in radians. CP11, etc., are the orthotropic elastic constants in the primed coordinate system. C11(M), etc., are the anisotropic elastic constants for the Mth lamina in the x, y, z coordinate system. AL1P(M), etc., are the coefficients of thermal expansion in the primed coordinate system and AL1(M) are those coefficients in the x, y, z coordinate system, both the the Mth lamina. Finally, the subroutine MATCON, which calculates the laminate MATerial CONstants, is called.

Part III. Part III calculates the coefficient matrix for the difference equations. The loops D0 100 and D0 101 count through the mesh node-by-node. D0 3000 zeroes out the A-matrix.

The logic that associates the various field conditions with each node and correctly fills out the A-matrix is contained in D0 102. First the node I, J is tested to determine the proper layer number, M. Then the node is checked to see if it lies on a boundary, along J equals a constant, or lies at some select position (in this case, IMID or JMID). If it does, the program is routed to the statement number that contains the non-zero matrix elements satisfying the conditions imposed at this node. Should the node not lie at any of these preselected locations, the program passes through the IF statements on J to statement number 193, which initiates a series of checks to see if the node lies on selected values of I. These values include the boundaries I = 1 or I = LAW, the changes in

nodal spacing  $I = FSW1$  or  $I = FSW2$ , and all points in the region between  $FSW1$  and  $FSW2$ . Should the node not lie at any of these locations, the program passes through the IF statements on  $I$  and evaluates the non-zero coefficients for the only remaining possibility, the equilibrium terms for a square mesh.

When a node does lie on some select location, say  $J$  equals  $LAT$ , then the logic in that statement series, say the series starting from statement number 202, guides the program through the checks on selected values of  $I$  in a fashion similar to that above. The logic is easily understood by reading directly from the listing.

Upon reaching statement number 102, the A-matrix ( $3 \times JQMAX$ ) is full. The elements of the A-matrix lying within the bandwidth are then stored in the banded matrix  $AX$ . The loops D0 100 or D0 101 then continue for the next node, if any. The previous A-matrix is destroyed and regenerated for the new node until the loop D0 100 is satisfied.

At rewind 9, the matrix  $AX$  and the load vector  $X$  are stored for later use. The loop D0 107 stores the load vector  $X(I)$  in  $AX(I, NBD)$ . Then a series of WRITE statements (listed as comments) will output the coefficient matrix  $AX$  and load vector  $X$  should they be desired. Finally, the solver routine,  $TRMSTR$ , is called.<sup>7</sup>

Part IV. Part IV outputs the functional displacements and provides an accuracy check. Just below statement number 4006, the STOP 1 statement will terminate the program if the coefficient matrix  $AX$  is singular. (Such an occurrence probably indicates an error.) The loop D0 108 stores the solution vector  $AX(I, 1)$  in  $X(I)$ . Then the original values of the matrix  $AX$  and load vector  $X$  are read back into the  $AX$  array and  $R$  vector, respectively.

The loops D0 11 and D0 12 output the values for the functions  $U(y, z)$ ,  $V(y, z)$ , and  $W(y, z)$  which occur in the displacements  $u$ ,  $v$ , and  $w$ , respectively.

The series of statements from the one above 9950 to 9990 outputs the accuracy results. These results provide the difference between the original load vector, now stored in the  $R$ -array, with the calculated load vector, which is found by substituting the appropriate solution vectors,  $X(I)$ , into each matrix equation. In addition to giving the accuracy of each equation, an average accumulated accuracy is provided.

Part V. Part V outputs the strains and stresses. The logic is similar to that in Part III. Knowing that the finite-difference relations for the strains differ for various mesh locations, the strains are split into terms dependent upon the value of  $I$  and terms dependent upon the value of  $J$ . The strain  $SX$ , which represents  $\epsilon_x$ , depends upon neither the value of  $I$  nor the value of  $J$  and is determined prior to any logical branching.

---

7. Actually the  $AX$ -matrix stores a transposed A-matrix; i.e., instead of storing row elements crosswise or in a row, they are stored in the  $AX$ -matrix vertically or in a column. The result is a drastic reduction in "wall-time" on the IBM 370. This necessitated a slight revision in the solver routine,  $TRIMSS$ , as written by Billy Gibbs, U.S. Army, Redstone Arsenal [14]. So here it is called  $TRMSTR$  or  $TRIMSS$  transposed.

First, the node is checked to determine its location with respect to I, and I-dependent strains (or the partial strain, SYZI) are calculated. Then the loop D0 392 establishes the correct layer number, M, in order to check if J lies on the interface, INF(M). Upon determining the correct location of the node with respect to J, the J-dependent strains (or the partial strain, SYZJ) are calculated. Statement number 391 totals the partial strains to obtain SYZ. The stresses are then calculated in a straight forward manner using equation (1). Note that the stresses are calculated twice at interface nodes, once for the material below the interface and again for the material above.

Part VI. The subroutine MATCON calculates the MATerial CONstants  $C_j$ , BU, BV, and DV as defined earlier in the text.

Part VII. The subroutine MAMULT is a MAtrix MULTiplier and is easily understood from the listing.

Part VIII. The subroutine MATIN4 is a MATrix INversion routine which is described in Reference 14.

Part IX. The subroutine TRMSTR is the equation solver which is described in the listing.

Part X. The subroutine RITE is used to wRITE out a matrix or vector.

#### Program Listing

The complete listing of the program is contained in the following pages.

```

C  JOMAX IS THE NUMBER OF UNKNOWNS OR EQUATIONS TO BE SOLVED.          00000010
C  A IS A FULL MATRIX (3 X JQMAX) REPRESENTING EACH NODE.          00000020
C  AX IS THE BANDED MATRIX (NBAND+1 X JQMAX).          00000030
C  X IS THE LOAD VECTOR. AFTER TRIMSS X BECOMES THE SOLUTION VECTOR. 00000040
C
C  IF THE NUMBER OF LAYERS EXCEED 6, THE COMMON /MC/ AND DIMENSION (E11,00000060
C  E22, ETC.) STATEMENTS MUST BE REDIMENSIONED TO AGREE WITH LAT. 00000070
C  REMEMBER TO PLACE A COMMON /MC/ STATEMENT IN SUBROUTINE MATCON. 00000080
C
C  USE THE FOLLOWING ORDER FOR DATA CARDS          00000090
C
C  DATA CARD NO.          DATA          FORMAT 00000120
C  1          NLAY, LAT, LAW, FSW1, K          5I10  00000130
C  2          H          G12.5  00000140
C  3          E11, E22, E33, E12, E13, E23          8G12.5 00000150
C  4          NU12, NU13, NU23          8G12.5  00000160
C  5          ALPHA 1 PRIME, ALPHA 2 PRIME, ALPHA 3 PRIME          8G12.5  00000170
C  NOTE, REPEAT CARDS OF THE TYPE 3, 4, 5 FOR EACH ADDITIONAL LAYER 00000180
C  6          SXMAX, C3E          10G10.3 00000190
C  7          IRUN          5I10  00000200
C  8          THETA(1), THETA(2), THETA(3), ETC.          10G10.3 00000210
C  NOTE, REPEAT CARD 8 FOR EACH ADDITIONAL LAYER.          00000220
C
C
C 0001      INTEGER P, FSW1, FSW2          00000230
C 0002      DOUBLE PRECISION TEST, R, ERR, AVE, DT          00000240
C 0003      DOUBLE PRECISION AX, X          00000250
C 0004      DOUBLE PRECISION THETA, BETA          00000260
C 0005      DOUBLE PRECISION CM, CN, CM4, CN4, CM3N, CN3M, CM2, CN2, GNU21, 00000270
C 1          GNU31, GNU32, DET, CP11, CP22, CP33, CP12, CP13, 00000280
C 2          CP23, CP44, CP55, CP66          00000290
C
C 0006      DIMENSION AX(162,351),A(3,351), X(351), R(351)          00000300
C
C 0007      COMMON /MC/ C11(6),C12(6),C16(6),C22(6),C26(6),C66(6),C13(6), 00000310
C 1          C23(6),C36(6),C44(6),C45(6),C55(6),C33(6),AL1(6),AL2(6), 00000320
C 2          AL3(6),AL6(6),C2,C3,C3E,C4,BU,DU,BV,DV,H,SXMAX,NLAY,INF(6) 00000330
C
C 0008      DIMENSION E11(6),E22(6),E33(6),E12(6),E13(6),E23(6),GNU12(6), 00000340
C 1          GNU13(6),GNU23(6),THETA(6), AL1P(6), AL2P(6), AL3P(6) 00000350
C
C 0009      TEMP = 0.0          00000360
C
C 0010      WRITE(6,600)          00000370
C 0011      READ(5,601)NLAY,LAT,LAW,FSW1,K          00000380
C
C 0012      FSW2=LAW-FSW1+1          00000390
C 0013      JQMAX = 3*LAW*LAT          00000400
C 0014      IBW = 2*(3*LAT+1)          00000410
C 0015      IBW1 = IBW+1          00000420
C 0016      NBAND = 2*IBW+1          00000430
C
C 0017      WRITE(6,602)NLAY,LAT,LAW,FSW1,FSW2,K          00000440
C 0018      LAT1=LAT-1          00000450
C 0019      IMID = (LAW+1)/2          00000460
C 0020      JMID = (LAT+1)/2          00000470
C
C 0021      DO 501 M=1, NLAY          00000480

```

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MAIN

DATE = 75007

08/16/07

```

0022           INF(M)=1+M*LAT1/NLAY                      00000590
0023           WRITE(6,608)M,INF(M)                      00000600
0024           501 CONTINUE                                00000610
0025           C NOTE THAT INF(NLAY) EQUALS LAT AND IS NOT AN ACTUAL INTERFACE. 00000620
0026           C                                                 00000630
0027           READ(5,603) H                           00000640
0028           WRITE(6,607) H                           00000650
0029           HSQ = H**2                                00000660
0030           WRITE(6,604)                                00000670
0031           DD 500 M=1,NLAY                          00000680
0032           READ(5,603)E11(M),E22(M),E33(M),E12(M),E13(M),E23(M) 00000690
0033           READ(5,603)GNU12(M),GNU13(M),GNU23(M)      00000700
0034           WRITE(6,605) M, E11(M), E22(M), E33(M), E12(M), E13(M), E23(M), 00000710
0035           1 GNU12(M), GNU13(M), GNU23(M)          00000720
0036           READ(5,603)AL1P(M), AL2P(M), AL3P(M)      00000730
0037           500 CONTINUE                                00000740
0038           C                                                 00000750
0039           READ(5,606) SXMAX, C3E                      00000760
0040           READ(5,601) IRUN                           00000770
0041           DO 3001 M=1,NLAY                          00000780
0042           BETA = .0174532925199433D0*THETA(M)      00000790
0043           CM=DCOS(BETA)                            00000800
0044           CN=DSIN(BETA)                            00000810
0045           IF(DABS(CM).LT.1.E-08) CM = 0.          00000820
0046           IF(DABS(CN).LT.1.E-08) CN = 0.          00000830
0047           CM4=CM**4                                00000840
0048           CN4=CN**4                                00000850
0049           CM3N=CM**3*CN                           00000860
0050           CN3M=CN**3*CM                           00000870
0051           CM2=CM**2                                00000880
0052           CN2=CN**2                                00000890
0053           GNU21=GNU12(M)*E22(M)/E11(M)            00000900
0054           GNU31=GNU13(M)*E33(M)/E11(M)            00000910
0055           GNU32=GNU23(M)*E33(M)/E22(M)            00000920
0056           DET=1.-GNU12(M)*GNU21-GNU23(M)*GNU31-GNU13(M)*GNU31 00000930
0057           1-2.*GNU12(M)*GNU23(M)*GNU31            00000940
0058           CP11=E11(M)*(1.-GNU23(M)*GNU32)/DET    00000950
0059           CP22=E22(M)*(1.-GNU13(M)*GNU31)/DET    00000960
0060           CP33=E33(M)*(1.-GNU12(M)*GNU21)/DET    00000970
0061           CP12=E11(M)*(GNU21+GNU23(M)*GNU31)/DET 00000980
0062           CP13=E11(M)*(GNU31+GNU21*GNU32)/DET    00000990
0063           CP23=E22(M)*(GNU32+GNU12(M)*GNU31)/DET 00001000
0064           CP44=E23(M)                                00001010

```

```

0064      CP55=E13(M)                                00001170
0065      CP66=E12(M)                                00001180
0066      C11(M)=CM4*CP11+2.*CM2*CN2*CP12+CN4*CP22+4.*CM2*CN2*CP66 00001190
0067      C12(M)=CM2*CN2*CP11+(CM4+CN4)*CP12+CM2*CN2*CP22-CM2*CN2*4.*CP66 00001200
0068      .C16(M)=CM3N*CP11-(CM3N-CN3M)*CP12-CN3M*CP22-2.*(CM3N-CN3M)*CP66 00001210
0069      C22(M)=CN4*CP11+2.*CM2*CN2*CP12+CM4*CP22+4.*CM2*CN2*CP66 00001220
0070      C26(M)=CN3M*CP11-(CN3M-CM3N)*CP12-CM3N*CP22-2.*(CN3M-CM3N)*CP66 00001230
0071      C66(M)=CM2*CN2*CP11-2.*CM2*CN2*CP12+CM2*CN2*CP22+(CM2-CN2)**2*CP6600001240
0072      C13(M)=CM2*CP13+CN2*CP23                                00001250
0073      C23(M)=CN2*CP13+CM2*CP23                                00001260
0074      C36(M)=CM*CN*(CP13-CP23)                                00001270
0075      C44(M)=CM2*CP44+CN2*CP55                                00001280
0076      C45(M)=CM*CN*(CP55-CP44)                                00001290
0077      C55(M)=CN2*CP44+CM2*CP55                                00001300
0078      C33(M)=CP33                                00001310
0079      C                                00001320
0080      C                                00001330
0081      C*****CALCULATION OF THE COEF. OF THERMAL EXPANSION REFERRED TO X,Y,Z***** 00001340
0082      C                                00001350
0083      C*****CALCULATION OF THE COEF. OF THERMAL EXPANSION REFERRED TO X,Y,Z***** 00001360
0084      C                                00001370
0085      C*****CALCULATION OF THE COEF. OF THERMAL EXPANSION REFERRED TO X,Y,Z***** 00001380
0086      C                                00001390
0087      AL1(M)=CM2*AL1P(M)+CN2*AL2P(M)                                00001400
0088      AL2(M)=CN2*AL1P(M)+CM2*AL2P(M)                                00001410
0089      AL3(M)=AL3P(M)                                00001420
0090      AL6(M)=2.*CM*CN*(AL1P(M)-AL2P(M))                                00001430
0091      C                                00001440
0092      C      WRITE(6,620) M, C11(M), C12(M), C13(M), XX, XX, C16(M), CP11, 00001450
0093      C      1          CP12, CP13, XX, XX, C22(M), C23(M), XX, XX, 00001460
0094      C      2          C26(M), CP22, CP23, XX, XX, XX, C33(M), XX, XX, 00001470
0095      C      3          C36(M), CP33, XX, XX, XX, THETA(M), C44(M), C45(M), 00001480
0096      C      4          XX, CP44, XX, XX, C55(M), XX, CP55, XX, C66(M), CP66 00001490
0097      C      00001500
0098      3001 CONTINUE                                00001510
0099      C      00001520
0100      C      WRITE(6,611)                                00001530
0101      C      00001540
0102      C      DO 503 M=1,NLAY                                00001550
0103      C      WRITE(6,614) M, THETA(M), AL1(M), AL2(M), AL3(M), AL6(M), 00001560
0104      C      1          AL1P(M), AL2P(M), AL3P(M)                                00001570
0105      503 CONTINUE                                00001580
0106      C      00001590
0107      C      CALL MATCON                                00001600
0108      C      00001610
0109      C      00001620
0110      C*****CALCULATION OF THE COEFFICIENT MATRIX FOR THE DIFFERENCE EQUATIONS***** 00001630
0111      C      00001640
0112      C*****CALCULATION OF THE COEFFICIENT MATRIX FOR THE DIFFERENCE EQUATIONS***** 00001650
0113      C      00001660
0114      C      00001670
0115      C      00001680
0116      KJ1 = 1                                00001690
0117      KQ1 = KJ1 + 1                                00001700
0118      KQ2 = KJ1 + 2                                00001710
0119      C      00001720
0120      DO 100 I=1,LAW                                00001730
0121      DO 101 J=1, LAT                                00001740

```

```

C
0095      DO 3000 IM = KJ1, KQ2          00001750
0096      DO 3000 JM = 1, JQMAX        00001760
0097      A(IM,JM) = 0.                00001770
0098      3000 CONTINUE                 00001780
0099      C
0100      I1=I-1                      00001790
0101      I2=I-2                      00001800
0102      Z = (FLOAT(J)-(FLOAT(LAT)+1.)/2.)*H 00001810
0103      NODE = LAT*I1+J              00001820
0104      JJ1 = 3*(LAT*I1+J)-2        00001830
0105      JJ2 = 3*(LAT*I2+J)-2        00001840
0106      JJ3 = 3*(LAT*I2+J)-5        00001850
0107      JJ4 = 3*(LAT*I1+J)-2        00001860
0108      JJ5 = 3*(LAT*I1+J)+1        00001870
0109      JJ6 = 3*(LAT*I1+J)+1        00001880
0110      JJ7 = 3*(LAT*I2+J)+1        00001890
0111      JJ8 = 3*(LAT*I1+J)-5        00001900
0112      JJ9 = 3*(LAT*I1+J)-5        00001910
0113      JJ10 = 3*(LAT*I1+J)-8       00001920
0114      JJ11 = 3*(LAT*(I1+1)+J)-2   00001930
0115      JJ12 = 3*(LAT*I1+J)+4       00001940
0116      JJ13 = 3*(LAT*(I-3)+J)-2   00001950
0117      C
0118      JQ1 = JJ1+1                  00001960
0119      JQ2 = JJ1+2                  00001970
0120      C
0121      DO 102 M=1, NLAY           00001980
0122      IF(M.EQ.1.AND.J.GT.INF(1)) GO TO 102 00001990
0123      IF(M.EQ.1) GO TO 192        00002000
0124      IF(J.LE.INF(M-1).OR.J.GT.INF(M)) GO TO 102 00002010
0125      192 IF(J.EQ.1) GO TO 200    00002020
0126      IF(I.EQ.IMID.AND.J.EQ.JMID) GO TO 203 00002030
0127      IF(I.EQ.IMID+1.AND.J.EQ.JMID) GO TO 203 00002040
0128      IF(J.EQ.LAT) GO TO 202      00002050
0129      IF(J.EQ.INF(M)) GO TO 201    00002060
0130      C
0131      C SHOULD J EQUAL NONE OF THE ABOVE, CONTINUE ON BELOW TO STATEMENT 193 00002070
0132      C
0133      193 IF(I.EQ.1) GO TO 194    00002080
0134      IF(I.EQ.FSW1.OR.I.EQ.FSW2) GO TO 195 00002090
0135      IF(I.LT.FSW2.AND.I.GT.FSW1) GO TO 197 00002100
0136      IF(I.EQ.LAW) GO TO 198      00002110
0137      C
0138      C EQUILIBRIUM MATRIX TERMS FOR A SQUARE MESH, H1=H2=H3=H 00002120
0139      C
0140      A(KJ1,JJ1) = -8.*(C66(M)+C55(M)) 00002130
0141      A(KJ1,JJ2) = 4.*C66(M)            00002140
0142      A(KJ1,JJ4) = 4.*C66(M)            00002150
0143      A(KJ1,JJ6) = 4.*C55(M)            00002160
0144      A(KJ1,JJ8) = 4.*C55(M)            00002170
0145      A(KJ1,JJ1+1) = -8.*(C26(M)+C45(M)) 00002180
0146      A(KJ1,JJ2+1) = 4.*C26(M)          00002190
0147      A(KJ1,JJ4+1) = 4.*C26(M)          00002200
0148      A(KJ1,JJ6+1) = 4.*C45(M)          00002210
0149      A(KJ1,JJ8+1) = 4.*C45(M)          00002220
0150      C
0151      C = C36(M)+C45(M)            00002230

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C
0142      A(KJ1,JJ3+2) = C          00002330
0143      A(KJ1,JJ5+2) = C          00002340
0144      A(KJ1,JJ7+2) = -C         00002350
0145      A(KJ1,JJ9+2) = -C         00002360
C
0146      X(JJ1) = 0.              00002370
C
0147      A(KQ1,JJ1) = -8.*(C26(M)+C45(M)) 00002380
0148      A(KQ1,JJ2) = 4.*C26(M)           00002390
0149      A(KQ1,JJ4) = 4.*C26(M)           00002400
0150      A(KQ1,JJ6) = 4.*C45(M)           00002410
0151      A(KQ1,JJ8) = 4.*C45(M)           00002420
0152      A(KQ1,JJ1+1) = -8.*(C22(M)+C44(M)) 00002430
0153      A(KQ1,JJ2+1) = 4.*C22(M)           00002440
0154      A(KQ1,JJ4+1) = 4.*C22(M)           00002450
0155      A(KQ1,JJ6+1) = 4.*C44(M)           00002460
0156      A(KQ1,JJ8+1) = 4.*C44(M)           00002470
C
0157      D = C23(M)+C44(M)           00002480
C
0158      A(KQ1,JJ3+2) = D          00002490
0159      A(KQ1,JJ5+2) = D          00002500
0160      A(KQ1,JJ7+2) = -D         00002510
0161      A(KQ1,JJ9+2) = -D         00002520
C
0162      X(JQ1) = 0.              00002530
C
0163      A(KQ2,JJ3) = C          00002540
0164      A(KQ2,JJ5) = C          00002550
0165      A(KQ2,JJ7) = -C         00002560
0166      A(KQ2,JJ9) = -C         00002570
0167      A(KQ2,JJ3+1) = D          00002580
0168      A(KQ2,JJ5+1) = D          00002590
0169      A(KQ2,JJ7+1) = -D         00002600
0170      A(KQ2,JJ9+1) = -D         00002610
0171      A(KQ2,JJ1+2) = -8.*(C44(M)+C33(M)) 00002620
0172      A(KQ2,JJ2+2) = 4.*C44(M)           00002630
0173      A(KQ2,JJ4+2) = 4.*C44(M)           00002640
0174      A(KQ2,JJ6+2) = 4.*C33(M)           00002650
0175      A(KQ2,JJ8+2) = 4.*C33(M)           00002660
C
0176      X(JQ2) = -4.*(C13(M)*C2 + C23(M)*DV + 2.*C36(M)*C4)*HSQ 00002670
0177      GO TO 102               00002680
C
C FREE SURFACE MATRIX TERMS FOR I=1 AND J NOT EQUAL TO 1, INF OR LAT 00002690
C
0178      194 A(KJ1,JJ1) = -3.*C66(M) 00002700
0179      A(KJ1,JJ4) = 4.*C66(M)       00002710
0180      A(KJ1,JJ11) = -C66(M)        00002720
0181      A(KJ1,JJ1+1) = -3.*C26(M)   00002730
0182      A(KJ1,JJ4+1) = 4.*C26(M)   00002740
0183      A(KJ1,JJ11+1) = -C26(M)    00002750
0184      A(KJ1,JJ6+2) = C36(M)      00002760
0185      A(KJ1,JJ8+2) = -C36(M)     00002770
C
0186      A(KQ1,JJ1) = -3.*C26(M)   00002780
0187      A(KQ1,JJ4) = 4.*C26(M)    00002790

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0188      A(KQ1,JJ11) = -C26(M)          00002910
0189      A(KQ1,JJ1+1) = -3.*C22(M)      00002920
0190      A(KQ1,JJ4+1) = 4.*C22(M)       00002930
01      A(KQ1,JJ11+1) = -C22(M)       00002940
01      A(KQ1,JJ6+2) = C23(M)         00002950
0193      A(KQ1,JJ8+2) = -C23(M)       00002960
00002970
C
0194      A(KQ2,JJ6) = C45(M)          00002980
0195      A(KQ2,JJ8) = -C45(M)        00002990
0196      A(KQ2,JJ6+1) = C44(M)        00003000
0197      A(KQ2,JJ8+1) = -C44(M)       00003010
0198      A(KQ2,JJ1+2) = -3.*C44(M)    00003020
0199      A(KQ2,JJ4+2) = 4.*C44(M)     00003030
0200      A(KQ2,JJ11+2) = -C44(M)      00003040
00003050
C
0201      CY1 = C12(M)*C3 + C22(M)*BV + C26(M)*BU 00003060
0202      CY2 = C12(M)*C2 + C22(M)*DV + 2.*C26(M)*C4 00003070
0203      CXY1 = C16(M)*C3 + C26(M)*BV + C66(M)*BU 00003080
0204      CXY2 = C16(M)*C2 + C26(M)*DV + 2.*C66(M)*C4 00003090
00003100
C
0205      X(JJ1) = -2.*H*(CXY1 + CXY2*Z) 00003110
0206      X(JQ1) = -2.*H*(CY1 + CY2*Z) 00003120
0207      X(JQ2) = 0. 00003130
0208      GO TO 102 00003140
00003150
C
0209      195 H1 = H 00003160
0210      H2 = FLOAT(K)*H 00003170
0211      H3 = H 00003180
00003190
C
0212      IF(I.NE.FSW2) GO TO 196 00003200
0213      H1 = FLOAT(K)*H 00003210
0214      H2 = H 00003220
00003230
C
0215      196 CONTINUE 00003240
0216      HH = H2/H1 00003250
0217      HR = HH/(1.+HH) 00003260
0218      HH1 = H1/H3 00003270
0219      HH2 = H2/H3 00003280
0220      HH3 = H1*H2 00003290
0221      HMU = HH1*HH2 00003300
0222      GO TO 199 00003310
00003320
C
0223      197 H1 = FLOAT(K)*H 00003330
0224      H2 = H1 00003340
0225      H3 = H 00003350
0226      GO TO 196 00003360
00003370
C
C FREE SURFACE MATRIX TERMS FOR I=LAW AND J NOT EQUAL TO 1, INF OR LAT 00003380
C
0227      198 A(KJ1,JJ1) = 3.*C66(M) 00003390
0228      A(KJ1,JJ2) = -4.*C66(M) 00003400
0229      A(KJ1,JJ13) = C66(M) 00003410
0230      A(KJ1,JJ1+1) = 3.*C26(M) 00003420
0231      A(KJ1,JJ2+1) = -4.*C26(M) 00003430
0232      A(KJ1,JJ13+1) = C26(M) 00003440
0233      A(KJ1,JJ6+2) = C36(M) 00003450
0234      A(KJ1,JJ8+2) = -C36(M) 00003460
00003470
C
00003480

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0235      A(KQ1,JJ1) = 3.*C26(M)          00003490
0236      A(KQ1,JJ2) = -4.*C26(M)         00003500
0237      A(KQ1,JJ13) = C26(M)           00003510
0238      A(KQ1,JJ1+1) = 3.*C22(M)         00003520
0239      A(KQ1,JJ2+1) = -4.*C22(M)        00003530
0240      A(KQ1,JJ13+1) = C22(M)           00003540
0241      A(KQ1,JJ6+2) = C23(M)           00003550
0242      A(KQ1,JJ8+2) = -C23(M)          00003560
0243      C
0244      A(KQ2,JJ6) = C45(M)           00003580
0245      A(KQ2,JJ8) = -C45(M)          00003590
0246      A(KQ2,JJ6+1) = C44(M)           00003600
0247      A(KQ2,JJ8+1) = -C44(M)          00003610
0248      A(KQ2,JJ1+2) = 3.*C44(M)         00003620
0249      A(KQ2,JJ2+2) = -4.*C44(M)        00003630
0249      A(KQ2,JJ13+2) = C44(M)          00003640
0250      C
0251      CY1 = C12(M)*C3 + C22(M)*BV + C26(M)*BU 00003660
0252      CY2 = C12(M)*C2 + C22(M)*DV + 2.*C26(M)*C4 00003670
0253      CXY1 = C16(M)*C3 + C26(M)*BV + C66(M)*BU 00003680
0253      CXY2 = C16(M)*C2 + C26(M)*DV + 2.*C66(M)*C4 00003690
0254      C
0255      X(JJ1) = -2.*H*(CXY1 + CXY2*Z) 00003710
0256      X(JQ1) = -2.*H*(CY1 + CY2*Z) 00003720
0256      X(JQ2) = 0. 00003730
0257      GO TO 102 00003740
0258      C
0259      C EQUILIBRIUM MATRIX TERMS FOR A VARIABLE MESH, H1, H2 , H3 INDEPENDENT 00003760
0260      C
0261      199 A(KJ1,JJ1) = -2.*(C66(M)+HMU*C55(M)) 00003780
0262      A(KJ1,JJ2) = 2.*HR*C66(M) 00003790
0263      A(KJ1,JJ4) = 2.*C66(M)/(1.+HH) 00003800
0264      A(KJ1,JJ6) = HMU*C55(M) 00003810
0265      A(KJ1,JJ8) = HMU*C55(M) 00003820
0266      A(KJ1,JJ1+1) = -2.*(C26(M)+HMU*C45(M)) 00003830
0267      A(KJ1,JJ2+1) = 2.*HR*C26(M) 00003840
0268      A(KJ1,JJ4+1) = 2.*C26(M)/(1.+HH) 00003850
0269      A(KJ1,JJ6+1) = HMU*C45(M) 00003860
0270      A(KJ1,JJ8+1) = HMU*C45(M) 00003870
0271      C
0272      C = HH1*HR*(C36(M)+C45(M))/2. 00003880
0273      C
0274      A(KJ1,JJ3+2) = C 00003890
0275      A(KJ1,JJ5+2) = C 00003910
0276      A(KJ1,JJ7+2) = -C 00003920
0277      A(KJ1,JJ9+2) = -C 00003930
0278      C
0279      A(KQ1,JJ1) = -2.*(C26(M)+HMU*C45(M)) 00003960
0280      A(KQ1,JJ2) = 2.*HR*C26(M) 00003970
0281      A(KQ1,JJ4) = 2.*C26(M)/(1.+HH) 00003980
0282      A(KQ1,JJ6) = HMU*C45(M) 00003990
0283      A(KQ1,JJ8) = HMU*C45(M) 00004000
0284      A(KQ1,JJ1+1) = -2.*(C22(M)+HMU*C44(M)) 00004010
0285      A(KQ1,JJ2+1) = 2.*HR*C22(M) 00004020
0286      A(KQ1,JJ4+1) = 2.*C22(M)/(1.+HH) 00004030
0287      A(KQ1,JJ6+1) = HMU*C44(M) 00004040
0288      A(KQ1,JJ8+1) = HMU*C44(M) 00004050
0289      C

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0283      D = HH1*HR*(C23(M)+C44(M))/2.          00004070
          C
0284      A(KQ1,JJ3+2) = D                      00004080
0285      A(KQ1,JJ5+2) = D                      00004090
0286      A(KQ1,JJ7+2) = -D                     00004100
0287      A(KQ1,JJ9+2) = -D                     00004110
          C
0288      A(KQ2,JJ3) = C                      00004120
0289      A(KQ2,JJ5) = C                      00004130
0290      A(KQ2,JJ7) = -C                     00004140
0291      A(KQ2,JJ9) = -C                     00004150
0292      A(KQ2,JJ3+1) = D                     00004160
0293      A(KQ2,JJ5+1) = D                     00004170
0294      A(KQ2,JJ7+1) = -D                    00004180
0295      A(KQ2,JJ9+1) = -D                    00004190
0296      A(KQ2,JJ1+2) = -2.*{C44(M)+HMU*C33(M)} 00004200
0297      A(KQ2,JJ2+2) = 2.*HR*C44(M)        00004210
0298      A(KQ2,JJ4+2) = 2.*C44(M)/(1.+HH)    00004220
0299      A(KQ2,JJ6+2) = HMU*C33(M)        00004230
0300      A(KQ2,JJ8+2) = HMU*C33(M)        00004240
          C
0301      X(JJ1) = 0.                      00004250
0302      X(JQ1) = 0.                      00004260
0303      X(JQ2) = -HH3*(C13(M)*C2 + C23(M)*DV + 2.*C36(M)*C4) 00004270
0304      GO TO 102
          C
0305      200 IF(I.EQ.1) GO TO 210          00004280
0306      IF(I.EQ.LAW) GO TO 211
          C
C FREE SURFACE MATRIX TERMS FOR I BETWEEN 1 AND LAW AND J=1.
          C
0307      A(KJ1,JJ1) = -3.*C55(M)        00004290
0308      A(KJ1,JJ6) = 4.*C55(M)        00004300
0309      A(KJ1,JJ12) = -C55(M)        00004310
          C
0310      A(KJ1,JJ1+1) = -3.*C45(M)        00004320
0311      A(KJ1,JJ6+1) = 4.*C45(M)        00004330
0312      A(KJ1,JJ12+1) = -C45(M)        00004340
          C
0313      A(KQ1,JJ1) = -3.*C45(M)        00004350
0314      A(KQ1,JJ6) = 4.*C45(M)        00004360
0315      A(KQ1,JJ12) = -C45(M)        00004370
          C
0316      A(KQ1,JJ1+1) = -3.*C44(M)        00004380
0317      A(KQ1,JJ6+1) = 4.*C44(M)        00004390
0318      A(KQ1,JJ12+1) = -C44(M)        00004400
          C
0319      A(KQ2,JJ1+2) = -3.*C33(M)        00004410
0320      A(KQ2,JJ6+2) = 4.*C33(M)        00004420
0321      A(KQ2,JJ12+2) = -C33(M)        00004430
          C
0322      CZ1 = C13(M)*C3 + C23(M)*BV + C36(M)*BU 00004440
0323      CZ2 = C13(M)*C2 + C23(M)*DV + 2.*C36(M)*C4 00004450
          C
0324      X(JJ1) = 0.                      00004460
0325      X(JQ1) = 0.                      00004470
0326      X(JQ2) = -2.*H*(CZ1 + CZ2*Z)    00004480
          C

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0327      IF(I.EQ.FSW1) GO TO 206          00004650
0328      IF(I.EQ.FSW2) GO TO 206          00004660
0329      IF(I.GT.FSW1.AND.I.LT.FSW2) GO TO 209          00004670
C
C IF I IS BETWEEN 1 AND FSW1 OR BETWEEN FSW2 AND LAW, CONTINUE BELOW 00004680
C
0330      A(KJ1,JJ2+2) = -C45(M)          00004700
0331      A(KJ1,JJ4+2) = C45(M)          00004710
C
0332      A(KQ1,JJ2+2) = -C44(M)          00004720
0333      A(KQ1,JJ4+2) = C44(M)          00004730
C
0334      A(KQ2,JJ2) = -C36(M)          00004740
0335      A(KQ2,JJ4) = C36(M)          00004750
0336      A(KQ2,JJ2+1) = -C23(M)          00004760
0337      A(KQ2,JJ4+1) = C23(M)          00004770
0338      GO TO 102          00004780
C
C CASE WHERE I=FSW1 OR FSW2 AND J=1          00004790
C
0339      206 XK = FLOAT(K)          00004800
0340      D1 = 2.*(XK-1.)/XK          00004810
0341      D2 = 2.*XK/(XK+1.)          00004820
0342      D3 = 2./((XK+1.)*XK)          00004830
C
0343      IF(I.EQ.FSW2) GO TO 207          00004840
C
0344      A(KJ1,JJ1+2) = D1*C45(M)          00004850
0345      A(KJ1,JJ2+2) = -D2*C45(M)          00004860
0346      A(KJ1,JJ4+2) = D3*C45(M)          00004870
C
0347      A(KQ1,JJ1+2) = D1*C44(M)          00004880
0348      A(KQ1,JJ2+2) = -D2*C44(M)          00004890
0349      A(KQ1,JJ4+2) = D3*C44(M)          00004900
C
0350      A(KQ2,JJ1) = D1*C36(M)          00004910
0351      A(KQ2,JJ2) = -D2*C36(M)          00004920
0352      A(KQ2,JJ4) = D3*C36(M)          00004930
C
0353      A(KQ2,JJ1+1) = D1*C23(M)          00004940
0354      A(KQ2,JJ2+1) = -D2*C23(M)          00004950
0355      A(KQ2,JJ4+1) = D3*C23(M)          00004960
0356      GO TO 102          00004970
C
0357      207 A(KJ1,JJ1+2) = -D1*C45(M)          00004980
0358      A(KJ1,JJ2+2) = -D3*C45(M)          00004990
0359      A(KJ1,JJ4+2) = D2*C45(M)          00005000
C
0360      A(KQ1,JJ1+2) = -D1*C44(M)          00005010
0361      A(KQ1,JJ2+2) = -D3*C44(M)          00005020
0362      A(KQ1,JJ4+2) = D2*C44(M)          00005030
C
0363      A(KQ2,JJ1) = -D1*C36(M)          00005040
0364      A(KQ2,JJ2) = -D3*C36(M)          00005050
0365      A(KQ2,JJ4) = D2*C36(M)          00005060
C
0366      A(KQ2,JJ1+1) = -D1*C23(M)          00005070
0367      A(KQ2,JJ2+1) = -D3*C23(M)          00005080
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0368      A(KQ2,JJ4+1) = D2*C23(M)          00005230
0369      GO TO 102                         00005240
      C
      C CASE WHERE I IS BETWEEN FSW1 AND FSW2 AND J=1
      C
0370      209 XK = FLOAT(K)                  00005250
0371      A(KJ1,JJ2+2) = -C45(M)/XK        00005260
0372      A(KJ1,JJ4+2) = C45(M)/XK        00005270
      C
0373      A(KQ1,JJ2+2) = -C44(M)/XK        00005280
0374      A(KQ1,JJ4+2) = C44(M)/XK        00005290
      C
0375      A(KQ2,JJ2) = -C36(M)/XK        00005300
0376      A(KQ2,JJ4) = C36(M)/XK        00005310
      C
0377      A(KQ2,JJ2+1) = -C23(M)/XK        00005320
0378      A(KQ2,JJ4+1) = C23(M)/XK        00005330
0379      GO TO 102                         00005340
      C
      C FREE SURFACE MATRIX TERMS FOR I=J=1
      C
0380      210 A(KJ1,JJ1) = -3.*C66(M)      00005420
0381      A(KJ1,JJ4) = 4.*C66(M)          00005430
0382      A(KJ1,JJ11) = -C66(M)          00005440
0383      A(KJ1,JJ1+1) = -3.*C26(M)      00005450
0384      A(KJ1,JJ4+1) = 4.*C26(M)          00005460
0385      A(KJ1,JJ11+1) = -C26(M)          00005470
0386      A(KJ1,JJ1+2) = -3.*C36(M)      00005480
0387      A(KJ1,JJ6+2) = 4.*C36(M)          00005490
0388      A(KJ1,JJ12+2) = -C36(M)          00005500
      C
0389      A(KQ1,JJ1) = -3.*C26(M)          00005510
0390      A(KQ1,JJ4) = 4.*C26(M)          00005520
0391      A(KQ1,JJ11) = -C26(M)          00005530
0392      A(KQ1,JJ1+1) = -3.*C22(M)      00005540
0393      A(KQ1,JJ4+1) = 4.*C22(M)          00005550
0394      A(KQ1,JJ11+1) = -C22(M)          00005560
0395      A(KQ1,JJ1+2) = -3.*C23(M)      00005570
0396      A(KQ1,JJ6+2) = 4.*C23(M)          00005580
0397      A(KQ1,JJ12+2) = -C23(M)          00005590
      C
0398      A(KQ2,JJ1) = -3.*C45(M)          00005600
0399      A(KQ2,JJ6) = 4.*C45(M)          00005610
0400      A(KQ2,JJ12) = -C45(M)          00005620
0401      A(KQ2,JJ1+1) = -3.*C44(M)      00005630
0402      A(KQ2,JJ6+1) = 4.*C44(M)          00005640
0403      A(KQ2,JJ12+1) = -C44(M)          00005650
0404      A(KQ2,JJ1+2) = -3.*C44(M)      00005660
0405      A(KQ2,JJ4+2) = 4.*C44(M)          00005670
0406      A(KQ2,JJ11+2) = -C44(M)          00005680
      C
0407      CY1 = C12(M)*C3 + C22(M)*BV + C26(M)*BU 00005690
0408      CY2 = C12(M)*C2 + C22(M)*DV + 2.*C26(M)*C4 00005700
0409      CXY1 = C16(M)*C3 + C26(M)*BV + C66(M)*BU 00005710
0410      CXY2 = C16(M)*C2 + C26(M)*DV + 2.*C66(M)*C4 00005720
      C
0411      X(JJ1) = -2.*H*(CXY1 + CXY2*Z) 00005730
0412      X(JQ1) = -2.*H*(CY1 + CY2*Z) 00005740
                                         00005750
                                         00005760
                                         00005770
                                         00005780
                                         00005790
                                         00005800

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0413      X(JQ2) = 0.          00005810
0414      GO TO 102          00005820
C
C FREE SURFACE MATRIX TERMS FOR J=1 AND I=LAW          00005830
C
0415      211 A(KJ1,JJ1) = 3.*C66(M)          00005840
0416      A(KJ1,JJ2) = -4.*C66(M)          00005850
0417      A(KJ1,JJ13) = C6(M)          00005860
0418      A(KJ1,JJ1+1) = 3.*C26(M)          00005870
0419      A(KJ1,JJ2+1) = -4.*C26(M)          00005880
0420      A(KJ1,JJ13+1) = C26(M)          00005890
0421      A(KJ1,JJ1+2) = -3.*C36(M)          00005900
0422      A(KJ1,JJ6+2) = 4.*C36(M)          00005910
0423      A(KJ1,JJ12+2) = -C36(M)          00005920
C
0424      A(KQ1,JJ1) = 3.*C26(M)          00005930
0425      A(KQ1,JJ2) = -4.*C26(M)          00005940
0426      A(KQ1,JJ13) = C26(M)          00005950
0427      A(KQ1,JJ1+1) = 3.*C22(M)          00005960
0428      A(KQ1,JJ2+1) = -4.*C22(M)          00005970
0429      A(KQ1,JJ13+1) = C22(M)          00005980
0430      A(KQ1,JJ1+2) = -3.*C23(M)          00005990
0431      A(KQ1,JJ6+2) = 4.*C23(M)          00006000
0432      A(KQ1,JJ12+2) = -C23(M)          00006010
C
0433      A(KQ2,JJ1) = -3.*C45(M)          00006020
0434      A(KQ2,JJ6) = 4.*C45(M)          00006030
0435      A(KQ2,JJ12) = -C45(M)          00006040
0436      A(KQ2,JJ1+1) = -3.*C44(M)          00006050
0437      A(KQ2,JJ6+1) = 4.*C44(M)          00006060
0438      A(KQ2,JJ12+1) = -C44(M)          00006070
0439      A(KQ2,JJ1+2) = 3.*C44(M)          00006080
0440      A(KQ2,JJ2+2) = -4.*C44(M)          00006090
0441      A(KQ2,JJ13+2) = C44(M)          00006100
C
0442      CY1 = C12(M)*C3 + C22(M)*BV + C26(M)*BU          00006110
0443      CY2 = C12(M)*C2 + C22(M)*DV + 2.*C26(M)*C4          00006120
0444      CXYY1 = C16(M)*C3 + C26(M)*BV + C66(M)*BU          00006130
0445      CXYY2 = C16(M)*C2 + C26(M)*DV + 2.*C66(M)*C4          00006140
C
0446      X(JJ1) = -2.*H*(CXYY1 + CXYY2*Z)          00006150
0447      X(JQ1) = -2.*H*(CY1 + CY2*Z)          00006160
0448      X(JQ2) = 0.          00006170
0449      GO TO 102          00006180
C
0450      201 P = M+1          00006190
0451      IF(I.EQ.1) GO TO 220          00006200
0452      IF(I.EQ.FSW1) GO TO 221          00006210
0453      IF(I.LT.FSW2.AND.I.GT.FSW1) GO TO 222          00006220
0454      IF(I.EQ.FSW2) GO TO 221          00006230
0455      IF(I.EQ.LAW) GO TO 223          00006240
C
C MATRIX TERMS AT INTERFACE FOR I BETWEEN 1 AND FSW1 OR FSW2 AND LAW          00006250
C
0456      A(KJ1,JJ1) = 3.*(C55(M)+C55(P))          00006260
0457      A(KJ1,JJ6) = -4.*C55(P)          00006270
0458      A(KJ1,JJ8) = -4.*C55(M)          00006280
0459      A(KJ1,JJ10) = C55(M)          00006290

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0460      A(KJ1,JJ12) = C55(P)          00006390
          C
0461      A(KJ1,JJ1+1) = 3.*(C45(M)+C45(P)) 00006400
0462      A(KJ1,JJ6+1) = -4.*C45(P)        00006410
0463      A(KJ1,JJ8+1) = -4.*C45(M)        00006420
0464      A(KJ1,JJ10+1) = C45(M)          00006430
0465      A(KJ1,JJ12+1) = C45(P)          00006440
          C
0466      A(KJ1,JJ2+2) = C45(P)-C45(M)  00006450
0467      A(KJ1,JJ4+2) = C45(M)-C45(P)  00006460
          C
0468      A(KQ1,JJ1) = 3.*(C45(M)+C45(P)) 00006470
0469      A(KQ1,JJ6) = -4.*C45(P)        00006480
0470      A(KQ1,JJ8) = -4.*C45(M)        00006490
0471      A(KQ1,JJ10) = C45(M)          00006500
0472      A(KQ1,JJ12) = C45(P)          00006510
          C
0473      A(KQ1,JJ1+1) = 3.*(C44(M)+C44(P)) 00006520
0474      A(KQ1,JJ6+1) = -4.*C44(P)        00006530
0475      A(KQ1,JJ8+1) = -4.*C44(M)        00006540
0476      A(KQ1,JJ10+1) = C44(M)          00006550
0477      A(KQ1,JJ12+1) = C44(P)          00006560
          C
0478      A(KQ1,JJ2+2) = C44(P)-C44(M)  00006570
0479      A(KQ1,JJ4+2) = C44(M)-C44(P)  00006580
          C
0480      A(KQ2,JJ2) = C36(P)-C36(M)  00006590
0481      A(KQ2,JJ4) = C36(M)-C36(P)  00006600
          C
0482      A(KQ2,JJ2+1) = C23(P)-C23(M)  00006610
0483      A(KQ2,JJ4+1) = C23(M)-C23(P)  00006620
          C
0484      A(KQ2,JJ1+2) = 3.*(C33(M)+C33(P)) 00006630
0485      A(KQ2,JJ6+2) = -4.*C33(P)        00006640
0486      A(KQ2,JJ8+2) = -4.*C33(M)        00006650
0487      A(KQ2,JJ10+2) = C33(M)          00006660
0488      A(KQ2,JJ12+2) = C33(P)          00006670
          C
0489      CZ1 = (C13(P)-C13(M))*C3 + (C23(P)-C23(M))*BV + (C36(P)-C36(M))*BU 00006680
0490      CZ2 = (C13(P)-C13(M))*C2+(C23(P)-C23(M))*DV+2.*((C36(P)-C36(M))*C4 00006690
          C
0491      X(JJ1) = 0.          00006700
0492      X(JQ1) = 0.          00006710
0493      X(JQ2) = 2.*H*(CZ1 + CZ2*Z) 00006720
0494      GO TO 102          00006730
          C
          C FREE SURFACE MATRIX TERMS AT ANY INTERFACE WHERE I=1 AND J=INF OR AT 00006740
          C THE FREE SURFACE POINT I=1, J=LAT 00006750
          C
0495      220 A(KJ1,JJ1) = -3.*C66(M)  00006760
0496      A(KJ1,JJ4) = 4.*C66(M)        00006770
0497      A(KJ1,JJ11) = -C66(M)        00006780
          C
0498      A(KJ1,JJ1+1) = -3.*C26(M)  00006790
0499      A(KJ1,JJ4+1) = 4.*C26(M)  00006800
0500      A(KJ1,JJ11+1) = -C26(M)  00006810
          C
0501      A(KJ1,JJ1+2) = 3.*C36(M)  00006820

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0502          A(KJ1,JJ8+2) = -4.*C36(M)          00006970
0503          A(KJ1,JJ10+2) = C36(M)          00006980
0504          C          A(KQ1,JJ1) = -3.*C26(M)          00006990
0505          A(KQ1,JJ4) = 4.*C26(M)          00007000
0506          A(KQ1,JJ11) = -C26(M)          00007020
0507          C          A(KQ1,JJ1+1) = -3.*C22(M)          00007030
0508          A(KQ1,JJ4+1) = 4.*C22(M)          00007040
0509          A(KQ1,JJ11+1) = -C22(M)          00007050
0510          C          A(KQ1,JJ1+2) = 3.*C23(M)          00007060
0511          A(KQ1,JJ8+2) = -4.*C23(M)          00007070
0512          A(KQ1,JJ10+2) = C23(M)          00007080
0513          C          A(KQ2,JJ1) = 3.*C45(M)          00007090
0514          A(KQ2,JJ8) = -4.*C45(M)          00007100
0515          A(KQ2,JJ10) = C45(M)          00007110
0516          C          A(KQ2,JJ1+1) = 3.*C44(M)          00007120
0517          A(KQ2,JJ8+1) = -4.*C44(M)          00007130
0518          A(KQ2,JJ10+1) = C44(M)          00007140
0519          C          A(KQ2,JJ1+2) = -3.*C44(M)          00007150
0520          A(KQ2,JJ4+2) = 4.*C44(M)          00007160
0521          A(KQ2,JJ11+2) = -C44(M)          00007170
0522          C          CY1 = C12(M)*C3 + C22(M)*BV + C26(M)*BU          00007180
0523          CY2 = C12(M)*C2 + C22(M)*DV + 2.*C26(M)*C4          00007190
0524          CXV1 = C16(M)*C3 + C26(M)*BV + C66(M)*BU          00007200
0525          CXV2 = C16(M)*C2 + C26(M)*DV + 2.*C66(M)*C4          00007210
0526          C          X(JJ1) = -2.*H*(CXY1 + CXY2*Z)          00007220
0527          X(JQ1) = -2.*H*(CY1 + CY2*Z)          00007230
0528          X(JQ2) = 0.          00007240
0529          GO TO 102          00007250
0530          C          C MATRIX TERMS AT THE INTERFACE FOR J=INF AND I=FSW1 OR I=FSW2          00007260
0531          C          221 XK = FLOAT(K)          00007270
0532          D1 = (XK-1.)/XK          00007280
0533          D2 = XK/(XK+1.)          00007290
0534          D3 = 1. / (XK+1.)*XK          00007300
0535          A(KJ1,JJ1) = 3.*(C55(M)+C55(P))          00007310
0536          A(KJ1,JJ6) = -4.*C55(P)          00007320
0537          A(KJ1,JJ8) = -4.*C55(M)          00007330
0538          A(KJ1,JJ10) = C55(M)          00007340
0539          A(KJ1,JJ12) = C55(P)          00007350
0540          C          A(KJ1,JJ1+1) = 3.*(C45(M)+C45(P))          00007360
0541          A(KJ1,JJ6+1) = -4.*C45(P)          00007370
0542          A(KJ1,JJ8+1) = -4.*C45(M)          00007380
0543          A(KJ1,JJ10+1) = C45(M)          00007390
0544          A(KJ1,JJ12+1) = C45(P)          00007400
0545          C          A(KQ1,JJ1) = 3.*(C45(M) + C45(P))          00007410
0546          A(KQ1,JJ6) = -4.*C45(P)          00007420

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0546      A(KQ1, JJ8) = -4.*C45(M)          00007550
0547      A(KQ1, JJ10) = C45(M)            00007560
0548      A(KQ1, JJ12) = C45(P)            00007570
      C
0549      A(KQ1, JJ1+1) = 3.*(C44(M)+C44(P)) 00007590
0550      A(KQ1, JJ6+1) = -4.*C44(P)        00007600
0551      A(KQ1, JJ8+1) = -4.*C44(M)        00007610
0552      A(KQ1, JJ10+1) = C44(M)          00007620
0553      A(KQ1, JJ12+1) = C44(P)          00007630
      C
0554      A(KQ2, JJ1+2) = 3.*(C33(M)+C33(P)) 00007650
0555      A(KQ2, JJ6+2) = -4.*C33(P)        00007660
0556      A(KQ2, JJ8+2) = -4.*C33(M)        00007670
0557      A(KQ2, JJ10+2) = C33(M)          00007680
0558      A(KQ2, JJ12+2) = C33(P)          00007690
      C
0559      CZ1 = (C13(P)-C13(M))*C3 + (C23(P)-C23(M))*BV + (C36(P)-C36(M))*BU 00007710
0560      CZ2 = (C13(P)-C13(M))*C2+(C23(P)-C23(M))*DV+2.* (C36(P)-C36(M))*C4 00007720
      C
0561      X(JJ1) = 0.                      00007740
0562      X(JQ1) = 0.                      00007750
0563      X(JQ2) = 2.*H*(CZ1 + CZ2*Z)      00007760
      C
0564      C=C45(M)-C45(P)                00007780
0565      D=C44(M)-C44(P)                00007790
0566      E=C23(M)-C23(P)                00007800
0567      CC=C36(M)-C36(P)               00007810
      C
0568      IF(I.EQ.FSW2) GO TO 227        00007830
      C
0569      A(KJ1, JJ1+2) = 2.*D1*C          00007850
0570      A(KJ1, JJ2+2) = -2.*D2*C        00007860
0571      A(KJ1, JJ4+2) = 2.*D3*C          00007870
      C
0572      A(KQ1, JJ1+2) = 2.*D1*D          00007890
0573      A(KQ1, JJ2+2) = -2.*D2*D        00007900
0574      A(KQ1, JJ4+2) = 2.*D3*D          00007910
      C
0575      A(KQ2, JJ1) = 2.*D1*CC         00007930
0576      A(KQ2, JJ2) = -2.*D2*CC        00007940
0577      A(KQ2, JJ4) = 2.*D3*CC         00007950
      C
0578      A(KQ2, JJ1+1) = 2.*D1*E          00007970
0579      A(KQ2, JJ2+1) = -2.*D2*E        00007980
0580      A(KQ2, JJ4+1) = 2.*D3*E          00007990
0581      GO TO 102                      00008000
      C
0582      227 A(KJ1, JJ1+2) = -2.*D1*C    00008020
0583      A(KJ1, JJ2+2) = -2.*D3*C        00008030
0584      A(KJ1, JJ4+2) = 2.*D2*C          00008040
      C
0585      A(KQ1, JJ1+2) = -2.*D1*D        00008060
0586      A(KQ1, JJ2+2) = -2.*D3*D        00008070
0587      A(KQ1, JJ4+2) = 2.*D2*D          00008080
      C
0588      A(KQ2, JJ1) = -2.*D1*CC        00008090
0589      A(KQ2, JJ2) = -2.*D3*CC        00008100
0590      A(KQ2, JJ4) = 2.*D2*CC          00008120

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      C
0591      A(KQ2,JJ1+1) = -2.*D1*E          00008130
0592      A(KQ2,JJ2+1) = -2.*D3*E          00008140
0593      A(KQ2,JJ4+1) = 2.*D2*E          00008150
0594      GO TO 102                      00008160
      C
      C MATRIX TERMS AT AN INTERFACE FOR J=INF AND I BETWEEN FSW1 AND FSW2 00008170
      C
0595      222 XK = FLOAT(K)                00008180
0596      A(KJ1,JJ1) = 3.*(C55(M)+C55(P)) 00008190
0597      A(KJ1,JJ6) = -4.*C55(P)          00008200
0598      A(KJ1,JJ8) = -4.*C55(M)          00008210
0599      A(KJ1,JJ10) = C55(M)            00008220
0600      A(KJ1,JJ12) = C55(P)            00008230
      C
0601      A(KJ1,JJ1+1) = 3.*(C45(M)+C45(P)) 00008240
0602      A(KJ1,JJ6+1) = -4.*C45(P)          00008250
0603      A(KJ1,JJ8+1) = -4.*C45(M)          00008260
0604      A(KJ1,JJ10+1) = C45(M)            00008270
0605      A(KJ1,JJ12+1) = C45(P)            00008280
      C
0606      A(KJ1,JJ2+2) = (C45(P)-C45(M))/XK 00008290
0607      A(KJ1,JJ4+2) = (C45(M)-C45(P))/XK 00008300
      C
0608      A(KQ1,JJ1) = 3.*(C45(M)+C45(P)) 00008310
0609      A(KQ1,JJ6) = -4.*C45(P)          00008320
0610      A(KQ1,JJ8) = -4.*C45(M)          00008330
0611      A(KQ1,JJ10) = C45(M)            00008340
0612      A(KQ1,JJ12) = C45(P)            00008350
      C
0613      A(KQ1,JJ1+1) = 3.*(C44(M)+C44(P)) 00008360
0614      A(KQ1,JJ6+1) = -4.*C44(P)          00008370
0615      A(KQ1,JJ8+1) = -4.*C44(M)          00008380
0616      A(KQ1,JJ10+1) = C44(M)            00008390
0617      A(KQ1,JJ12+1) = C44(P)            00008400
      C
0618      A(KQ1,JJ2+2) = (C44(P)-C44(M))/XK 00008410
0619      A(KQ1,JJ4+2) = (C44(M)-C44(P))/XK 00008420
      C
0620      A(KQ2,JJ2) = (C36(P)-C36(M))/XK 00008430
0621      A(KQ2,JJ4) = (C36(M)-C36(P))/XK 00008440
      C
0622      A(KQ2,JJ2+1) = (C23(P)-C23(M))/XK 00008450
0623      A(KQ2,JJ4+1) = (C23(M)-C23(P))/XK 00008460
      C
0624      A(KQ2,JJ1+2) = 3.*(C33(M)+C33(P)) 00008470
0625      A(KQ2,JJ6+2) = -4.*C33(P)          00008480
0626      A(KQ2,JJ8+2) = -4.*C33(M)          00008490
0627      A(KQ2,JJ10+2) = C33(M)            00008500
0628      A(KQ2,JJ12+2) = C33(P)            00008510
0629      X(JQ1) = 0.                      00008520
      C
0630      C21 = (C13(P)-C13(M))*C3 + (C23(P)-C23(M))*BV + (C36(P)-C36(M))*BU 00008530
0631      C22 = (C13(P)-C13(M))*C2+(C23(P)-C23(M))*DV+2.*{(C36(P)-C36(M))*C4} 00008540
      C
0632      X(JJ1) = 0.                      00008550
0633      X(JQ2) = 2.*H*(C21 + C22*Z)      00008560
0634      GO TO 102                      00008570
                                              00008580
                                              00008590
                                              00008600
                                              00008610
                                              00008620
                                              00008630
                                              00008640
                                              00008650
                                              00008660
                                              00008670
                                              00008680
                                              00008690
                                              00008700

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C          00008710
C FREE SURFACE MATRIX TERMS AT ANY INTERFACE WHERE I=LAW, J=INF OR AT 00008720
C THE FREE SURFACE POINT I=LAW, J=LAT 00008730
C          00008740
0635      223 A(KJ1,JJ1) = 3.*C66(M) 00008750
0636      A(KJ1,JJ2) = -4.*C66(M) 00008760
0637      A(KJ1,JJ13) = C66(M) 00008770
C          00008780
0638      A(KJ1,JJ1+1) = 3.*C26(M) 00008790
C          00008860
0644      A(KQ1,JJ1) = 3.*C26(M) 00008870
0645      A(KQ1,JJ2) = -4.*C26(M) 00008880
0646      A(KQ1,JJ13) = C26(M) 00008890
C          00008900
0647      A(KQ1,JJ1+1) = 3.*C22(M) 00008910
0648      A(KQ1,JJ2+1) = -4.*C22(M) 00008920
0649      A(KQ1,JJ13+1) = C22(M) 00008930
C          00008940
0650      A(KQ1,JJ1+2) = 3.*C23(M) 00008950
0651      A(KQ1,JJ8+2) = -4.*C23(M) 00008960
0652      A(KQ1,JJ10+2) = C23(M) 00008970
C          00008980
0653      A(KQ2,JJ1) = 3.*C45(M) 00008990
0654      A(KQ2,JJ8) = -4.*C45(M) 00009000
0655      A(KQ2,JJ10) = C45(M) 00009010
C          00009020
0656      A(KQ2,JJ1+1) = 3.*C44(M) 00009030
0657      A(KQ2,JJ8+1) = -4.*C44(M) 00009040
0658      A(KQ2,JJ10+1) = C44(M) 00009050
C          00009060
0659      A(KQ2,JJ1+2) = 3.*C44(M) 00009070
0660      A(KQ2,JJ2+2) = -4.*C44(M) 00009080
0661      A(KQ2,JJ13+2) = C44(M) 00009090
C          00009100
0662      CY1 = C12(M)*C3 + C22(M)*BV + C26(M)*BU 00009110
0663      CY2 = C12(M)*C2 + C22(M)*DV + 2.*C26(M)*C4 00009120
0664      CXY1 = C16(M)*C3 + C26(M)*BV + C66(M)*BU 00009130
0665      CXYY2 = C16(M)*C2 + C26(M)*DV + 2.*C66(M)*C4 00009140
C          00009150
0666      X(JJ1) = -2.*H*(CXYY1 + CXYY2*Z) 00009160
0667      X(JQ1) = -2.*H*(CY1 + CY2*Z) 00009170
0668      X(JQ2) = 0. 00009180
0669      GO TO 102 00009190
C          00009200
C MATRIX TERMS TO FIX THE RIGID TRANSLATIONS 00009210
C          00009220
0670      203 A(KJ1,JJ1) = 1.0 00009230
0671      A(KQ1,JJ1+1) = 1.0 00009240
0672      A(KQ2,JJ1+2) = 1.0 00009250
C          00009260
0673      X(JJ1) = 0. 00009270
0674      X(JQ1) = 0. 00009280
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0675      X(JQ2) = 0.          00009290
0676      GO TO 102          00009300
0677      C                   00009310
0678      202 IF(I.EQ.1) GO TO 220 00009320
      IF(I.EQ.LAW) GO TO 223 00009330
0679      C                   00009340
0680      C FREE SURFACE MATRIX TERMS FOR I BETWEEN 1 AND LAW AND J=LAT 00009350
0681      C                   00009360
0682      A(KJ1,JJ1) = 3.*C55(M) 00009370
0683      A(KJ1,JJ8) = -4.*C55(M) 00009380
0684      A(KJ1,JJ10) = C55(M) 00009390
0685      C                   00009400
0686      A(KJ1,JJ1+1) = 3.*C45(M) 00009410
0687      A(KJ1,JJ8+1) = -4.*C45(M) 00009420
0688      A(KJ1,JJ10+1) = C45(M) 00009430
0689      C                   00009440
0690      A(KQ1,JJ1) = 3.*C45(M) 00009450
0691      A(KQ1,JJ8) = -4.*C45(M) 00009460
0692      A(KQ1,JJ10) = C45(M) 00009470
0693      C                   00009480
0694      A(KQ1,JJ1+1) = 3.*C44(M) 00009490
0695      A(KQ1,JJ8+1) = -4.*C44(M) 00009500
0696      A(KQ1,JJ10+1) = C44(M) 00009510
0697      C                   00009520
0698      A(KQ2,JJ1+2) = 3.*C33(M) 00009530
0699      A(KQ2,JJ8+2) = -4.*C33(M) 00009540
0700      A(KQ2,JJ10+2) = C33(M) 00009550
0701      C                   00009560
0702      CZ1 = C13(M)*C3 + C23(M)*BV + C36(M)*BU 00009570
0703      CZ2 = C13(M)*C2 + C23(M)*DV + 2.*C36(M)*C4 00009580
0704      C                   00009590
0705      X(JJ1) = 0.          00009600
0706      X(JQ1) = 0.          00009610
0707      X(JQ2) = -2.*H*(CZ1 + CZ2*Z) 00009620
0708      C                   00009630
0709      IF(I.EQ.FSW1) GO TO 231 00009640
0710      IF(I.EQ.FSW2) GO TO 231 00009650
0711      IF(I.GT.FSW1.AND.I.LT.FSW2) GO TO 234 00009660
0712      C                   00009670
0713      C IF I IS BETWEEN 1 AND FSW1 OR BETWEEN FSW2 AND LAW, CONTINUE BELOW 00009680
0714      C                   00009690
0715      A(KJ1,JJ2+2) = -C45(M) 00009700
0716      A(KJ1,JJ4+2) = C45(M) 00009710
0717      C                   00009720
0718      A(KQ1,JJ2+2) = -C44(M) 00009730
0719      A(KQ1,JJ4+2) = C44(M) 00009740
0720      C                   00009750
0721      A(KQ2,JJ2) = -C36(M) 00009760
0722      A(KQ2,JJ4) = C36(M) 00009770
0723      C                   00009780
0724      A(KQ2,JJ2+1) = -C23(M) 00009790
0725      A(KQ2,JJ4+1) = C23(M) 00009800
0726      GO TO 102          00009810
0727      C                   00009820
0728      C CASE WHERE I=FSW1 OR FSW2 AND J=LAT 00009830
0729      C                   00009840
0730      231 XK = FLOAT(K) 00009850
0731      D1 = 2.*(XK-1.)/XK 00009860
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0713      D2 = 2.*XK/(XK+1.)
0714      D3 = 2./((XK+1.)*XK)
0715      C      IF(I1.EQ.FSW2) GO TO 232
0716      C      A(KJ1,JJ1+2) = D1*C45(M)
0717      C      A(KJ1,JJ2+2) = -D2*C45(M)
0718      C      A(KJ1,JJ4+2) = D3*C45(M)
0719      C      A(KQ1,JJ1+2) = D1*C44(M)
0720      C      A(KQ1,JJ2+2) = -D2*C44(M)
0721      C      A(KQ1,JJ4+2) = D3*C44(M)
0722      C      A(KQ2,JJ1) = D1*C36(M)
0723      C      A(KQ2,JJ2) = -D2*C36(M)
0724      C      A(KQ2,JJ4) = D3*C36(M)
0725      C      A(KQ2,JJ1+1) = D1*C23(M)
0726      C      A(KQ2,JJ2+1) = -D2*C23(M)
0727      C      A(KQ2,JJ4+1) = D3*C23(M)
0728      C      GO TO 102
0729      C      232 A(KJ1,JJ1+2) = -D1*C45(M)
0730      C      A(KJ1,JJ2+2) = -D3*C45(M)
0731      C      A(KJ1,JJ4+2) = D2*C45(M)
0732      C      A(KQ1,JJ1+2) = -D1*C44(M)
0733      C      A(KQ1,JJ2+2) = -D3*C44(M)
0734      C      A(KQ1,JJ4+2) = D2*C44(M)
0735      C      A(KQ2,JJ1) = -D1*C36(M)
0736      C      A(KQ2,JJ2) = -D3*C36(M)
0737      C      A(KQ2,JJ4) = D2*C36(M)
0738      C      A(KQ2,JJ1+1) = -D1*C23(M)
0739      C      A(KQ2,JJ2+1) = -D3*C23(M)
0740      C      A(KQ2,JJ4+1) = D2*C23(M)
0741      C      GO TO 102
0742      C      C CASE WHERE I IS BETWEEN FSW1 AND FSW2 AND J=LAT
0743      C      234 XK = FLOAT(K)
0744      C      A(KJ1,JJ2+2) = -C45(M)/XK
0745      C      A(KJ1,JJ4+2) = C45(M)/XK
0746      C      A(KQ1,JJ2+2) = -C44(M)/XK
0747      C      A(KQ1,JJ4+2) = C44(M)/XK
0748      C      A(KQ2,JJ2) = -C36(M)/XK
0749      C      A(KQ2,JJ4) = C36(M)/XK
0750      C      A(KQ2,JJ2+1) = -C23(M)/XK
0751      C      A(KQ2,JJ4+1) = C23(M)/XK
0751      C      102 CONTINUE
0751      C      C FORM THE NONSYMETRIC BANDED MATRIX AX
0751      C
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0752           IL = KJ1+3*(NODE-1)          00010450
0753           IN = IL+2                00010460
0754           C
0755           DO 103 IK=IL, IN          00010470
0755           II = IK-IL+1            00010480
0756           C
0756           DO 104 JK=1,NBAND        00010490
0757           JJ = IK+JK-IBW-1        00010500
0758           IF(IK.LE.IBW1) JJ = JK  00010510
0759           IF(JJ.GT.JQMAX) GO TO 105 00010520
0760           AX(JK,IK) = A(II,JJ)  00010530
0761           GO TO 104            00010540
0762           105 AX(JK,IK) = 0.0    00010550
0763           104 CONTINUE          00010560
0764           103 CONTINUE          00010570
0765           101 CONTINUE          00010580
0766           100 CONTINUE          00010590
0767           C
0767           REWIND 9              00010600
0768           WRITE(9) ((AX(J,I),J=1,NBAND),I=1,JQMAX) 00010610
0769           WRITE(9) (X(I),I=1,JQMAX) 00010620
0770           END FILE 9            00010630
0771           REWIND 9              00010640
0772           C
0772           NBD = NBAND+1          00010650
0773           DO 107 I=1, JQMAX        00010660
0774           AX(NBD,I) = X(I)      00010670
0775           107 CONTINUE          00010680
0776           C
0776           WRITE(6,4000)          00010690
C4000  FORMAT(1H1,' EQUATION', 35X, 'THE BANDED MATRIX TERMS AX(I,J)' //) 00010700
C     CALL RITE(1, JQMAX, NBD, JQMAX, NBD, AX) 00010710
C     WRITE(6,4003)          00010720
C4003  FORMAT(1H1, 45X, '*** THE LOAD VECTOR X(I) ***' //) 00010730
C     WRITE(6,4004) (X(I), I=1, JQMAX) 00010740
C4004  FORMAT(28(2X, 10D12.3 /)) 00010750
C
0776           CALL TRMSTR(AX, JQMAX, NBD, IBW, IBW, NBAND, DT, RT, ET) 00010760
0777           C
0777           WRITE(6,4006) ET, RT, DT 00010770
0778           4006 FORMAT(// ' ERROR CONDITION OF SOLVER ROUTINE IS ', F4.1, 5X, 00010780
0778           1 'RANK IS ', F6.1, 5X, 'DETERMINANT = ', G10.3) 00010790
0779           IF(ET.EQ.1.) STOP 1 00010800
0779           C
0780           DO 108 I=1,JQMAX        00010810
0781           X(I) = AX(1,I)          00010820
0782           108 CONTINUE          00010830
0783           C
0783           READ(9) ((AX(J,I),J=1,NBAND),I=1,JQMAX) 00010840
0784           READ(9) (R(I),I=1,JQMAX) 00010850
0784           C
0784           ***** 00010860
0784           C
0784           ***** 00010870
0784           C
0784           ***** 00010880
0784           C
0784           ***** 00010890
0784           C
0784           ***** 00010900
0784           C
0784           ***** 00010910
0784           C
0784           ***** 00010920
0784           C
0784           ***** 00010930
0784           C
0784           ***** 00010940
0784           C
0784           ***** 00010950
0784           C
0784           ***** 00010960
0784           C
0784           ***** 00010970
0784           C
0784           ***** 00010980
0784           C
0784           ***** 00010990
0784           C
0784           ***** 00011000
0784           C
0784           ***** 00011010
0784           C
0784           ***** 00011020

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0785      WRITE(6,650)                                     00011030
0786      J = 1                                         00011040
0787      DO 12 IK = 1, LAW                            00011050
0788      DO 11 JK = 1, LAT                            00011060
0789      WRITE(6,651) J, X(3*j-2), X(3*j-1), X(3*j)  00011070
0790      J = J+1                                       00011080
0791      11 CONTINUE
0792      WRITE(6,653)
0793      12 CONTINUE
C
0794      WRITE(6,9950)
0795      9950 FORMAT(1H1, 5X, 'EQUATION', 5X, '*** THE ACCURACY TEST, TEST-R(I) 00011140
1 ***', 10X, '*** THE AVERAGE ABSOLUTE ERROR ***' //)
0796      ERR = 0.0D0                                     00011150
0797      DO 9990 I=1,JQMAX                            00011160
0798      TEST = 0.0D0                                    00011170
0799      DO 9960 J=1,NBAND                            00011180
0800      IC = I+J-1BW-1                               00011190
0801      IF(I.LE.IBW1) IC = J                         00011200
0802      IF(IC.GT.JQMAX) GO TO 9970                  00011210
0803      TEST = TEST+AX(J,I)*X(IC)                   00011220
0804      9960 CONTINUE                                 00011230
0805      9970 TEST = TEST-R(I)                         00011240
0806      ERR = ERR+DABS(TEST)                         00011250
0807      AVE = ERR/I                                 00011260
0808      WRITE(6,9980) I, TEST, AVE                  00011270
0809      9980 FORMAT(5X, I4,10X, G15.8, 32X, G15.8)  00011280
0810      9990 CONTINUE                                 00011290
C
C **** CALCULATION OF THE STRAIN (S) AND STRESS (T) ****
C
0811      SXM = SXMAX * 1.E06                           00011310
0812      SXE = C3E * 1.E06                            00011320
0813      WRITE(6,670) SXM, SXE                         00011330
0814      WRITE(6,671)
0815      HR = 1./(2.*H)                               00011340
0816      XK = FLOAT(K)
C
0817      DO 399 I=1, LAW                            00011440
0818      DO 398 J=1, LAT                            00011450
C
0819      I1=I-1                                     00011460
0820      I2=I-2                                     00011470
0821      NODE = LAT*I1+J                            00011480
0822      JJ1 = 3*(LAT*I1+J)-2                      00011490
0823      JJ2 = 3*(LAT*I2+J)-2                      00011500
0824      JJ3 = 3*(LAT*I2+J)-5                      00011510
0825      JJ4 = 3*(LAT*I1+J)-2                      00011520
0826      JJ5 = 3*(LAT*I1+J)+1                      00011530
0827      JJ6 = 3*(LAT*I1+J)+1                      00011540
0828      JJ7 = 3*(LAT*I2+J)+1                      00011550
0829      JJ8 = 3*(LAT*I1+J)-5                      00011560
0830      JJ9 = 3*(LAT*I1+J)-5                      00011570
0831      JJ10 = 3*(LAT*I1+J)-8                     00011580
                                         00011590
                                         00011600

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0832      JJ11 = 3*(LAT*(I+1)+J)-2          00011610
0833      JJ12 = 3*(LAT*I1+J)+4          00011620
0834      JJ13 = 3*(LAT*(I-3)+J)-2          00011630
0835      C
0836      Z = (FLOAT(J)-(FLOAT(LAT)+1.)/2.)*H 00011640
0837      C
0838      IF(I.EQ.1) GO TO 385          00011650
0839      IF(I.EQ.LAW) GO TO 386          00011660
0840      IF(I.GT.FSW1.AND.I.LT.FSW2) GO TO 382 00011670
0841      IF(I.EQ.FSW1.OR.I.EQ.FSW2) GO TO 383 00011680
0842      C
0843      H1 = H          00011690
0844      H2 = H1          00011700
0845      GO TO 384          00011710
0846      C
0847      382 H1 = XK*H          00011720
0848      H2 = H1          00011730
0849      GO TO 384          00011740
0850      C
0851      H1 = XK*H          00011750
0852      H2 = H          00011760
0853      C
0854      383 H1 = H          00011770
0855      H2 = XK*H          00011780
0856      IF(I.EQ.FSW1) GO TO 384          00011790
0857      H1 = XK*H          00011800
0858      H2 = H          00011810
0859      C
0860      384 H12 = H1/H2          00011820
0861      H21 = H2/H1          00011830
0862      HRD = (H2-H1)/(H1*H2)          00011840
0863      HRS = 1./(H1+H2)          00011850
0864      C
0865      SY = HRS*(H12*X(JJ4+1)-H21*X(JJ2+1))+HRD*X(JJ1+1) + DV*Z + BV 00011860
0866      SXY = HRS*(H12*X(JJ4)-H21*X(JJ2)) + HRD*X(JJ1) + 2.*C4*Z + BU 00011870
0867      SYZI = HRS*(H12*X(JJ4+2)-H21*X(JJ2+2))+HRD*X(JJ1+2) 00011880
0868      GO TO 387          00011890
0869      C
0870      385 SY = HR*(4.*X(JJ4+1)-3.*X(JJ1+1)-X(JJ11+1)) + DV*Z + BV 00011900
0871      SXY = HR*(4.*X(JJ4)-3.*X(JJ1)-X(JJ11)) + 2.*C4*Z + BU 00011910
0872      SYZI = HR*(4.*X(JJ4+2)-3.*X(JJ1+2)-X(JJ11+2)) 00011920
0873      GO TO 387          00011930
0874      C
0875      386 SY = HR*(3.*X(JJ1+1)+X(JJ13+1)-4.*X(JJ2+1)) + DV*Z + BV 00011940
0876      SXY = HR*(3.*X(JJ1)+X(JJ13)-4.*X(JJ2)) + 2.*C4*Z + BU 00011950
0877      SYZI = HR*(3.*X(JJ1+2)+X(JJ13+2)-4.*X(JJ2+2)) 00011960
0878      C
0879      387 DO 392 M=1, NLAY          00011970
0880      IF(M.EQ.1.AND.J.GT.INF(1)) GO TO 392 00011980
0881      IF(M.EQ.1) GO TO 388          00011990
0882      IF(J.LE.INF(M-1).OR.J.GT.INF(M)) GO TO 392 00012000
0883      388 IF(J.EQ.1) GO TO 389          00012010
0884      IF(J.EQ.INF(M).OR.J.EQ.LAT) GO TO 390 00012020
0885      C
0886      SZ = HR*(X(JJ6+2)-X(JJ8+2))          00012030
0887      SYZJ = HR*(X(JJ6+1)-X(JJ8+1))          00012040
0888      SXZ = HR*(X(JJ6)-X(JJ8))          00012050
0889      GO TO 391          00012060
0890      C
0891      389 SZ = HR*(4.*X(JJ6+2)-3.*X(JJ1+2)-X(JJ12+2)) 00012070
0892      GO TO 391          00012080
0893      C
0894      390 DO 391 M=1, NLAY          00012090
0895      IF(M.EQ.1.AND.J.GT.INF(1)) GO TO 392 00012100
0896      IF(M.EQ.1) GO TO 388          00012110
0897      IF(J.LE.INF(M-1).OR.J.GT.INF(M)) GO TO 392 00012120
0898      391 IF(J.EQ.1) GO TO 392          00012130
0899      IF(J.EQ.INF(M).OR.J.EQ.LAT) GO TO 393 00012140
0900      C
0901      392 DO 393 M=1, NLAY          00012150
0902      IF(M.EQ.1.AND.J.GT.INF(1)) GO TO 394 00012160
0903      IF(M.EQ.1) GO TO 388          00012170
0904      IF(J.LE.INF(M-1).OR.J.GT.INF(M)) GO TO 394 00012180
0905      393 IF(J.EQ.1) GO TO 394          00012190
0906      IF(J.EQ.INF(M).OR.J.EQ.LAT) GO TO 395 00012200
0907      C
0908      394 DO 395 M=1, NLAY          00012210
0909      IF(M.EQ.1.AND.J.GT.INF(1)) GO TO 396 00012220
0910      IF(M.EQ.1) GO TO 388          00012230
0911      IF(J.LE.INF(M-1).OR.J.GT.INF(M)) GO TO 396 00012240
0912      395 IF(J.EQ.1) GO TO 396          00012250
0913      IF(J.EQ.INF(M).OR.J.EQ.LAT) GO TO 397 00012260
0914      C
0915      396 DO 397 M=1, NLAY          00012270
0916      IF(M.EQ.1.AND.J.GT.INF(1)) GO TO 398 00012280
0917      IF(M.EQ.1) GO TO 388          00012290
0918      IF(J.LE.INF(M-1).OR.J.GT.INF(M)) GO TO 398 00012300
0919      397 IF(J.EQ.1) GO TO 398          00012310
0920      IF(J.EQ.INF(M).OR.J.EQ.LAT) GO TO 399 00012320
0921      C
0922      398 DO 399 M=1, NLAY          00012330
0923      IF(M.EQ.1.AND.J.GT.INF(1)) GO TO 400 00012340
0924      IF(M.EQ.1) GO TO 388          00012350
0925      IF(J.LE.INF(M-1).OR.J.GT.INF(M)) GO TO 400 00012360
0926      399 IF(J.EQ.1) GO TO 400          00012370
0927      IF(J.EQ.INF(M).OR.J.EQ.LAT) GO TO 401 00012380
0928      C
0929      400 DO 401 M=1, NLAY          00012390
0930      IF(M.EQ.1.AND.J.GT.INF(1)) GO TO 402 00012400
0931      IF(M.EQ.1) GO TO 388          00012410
0932      IF(J.LE.INF(M-1).OR.J.GT.INF(M)) GO TO 402 00012420
0933      401 IF(J.EQ.1) GO TO 402          00012430
0934      IF(J.EQ.INF(M).OR.J.EQ.LAT) GO TO 403 00012440
0935      C
0936      402 DO 403 M=1, NLAY          00012450
0937      IF(M.EQ.1.AND.J.GT.INF(1)) GO TO 404 00012460
0938      IF(M.EQ.1) GO TO 388          00012470
0939      IF(J.LE.INF(M-1).OR.J.GT.INF(M)) GO TO 404 00012480
0940      403 IF(J.EQ.1) GO TO 404          00012490
0941      IF(J.EQ.INF(M).OR.J.EQ.LAT) GO TO 405 00012500
0942      C
0943      404 DO 405 M=1, NLAY          00012510
0944      IF(M.EQ.1.AND.J.GT.INF(1)) GO TO 406 00012520
0945      IF(M.EQ.1) GO TO 388          00012530
0946      IF(J.LE.INF(M-1).OR.J.GT.INF(M)) GO TO 406 00012540
0947      405 IF(J.EQ.1) GO TO 406          00012550
0948      IF(J.EQ.INF(M).OR.J.EQ.LAT) GO TO 407 00012560
0949      C
0950      406 DO 407 M=1, NLAY          00012570
0951      IF(M.EQ.1.AND.J.GT.INF(1)) GO TO 408 00012580
0952      IF(M.EQ.1) GO TO 388          00012590
0953      IF(J.LE.INF(M-1).OR.J.GT.INF(M)) GO TO 408 00012600
0954      407 IF(J.EQ.1) GO TO 408          00012610
0955      IF(J.EQ.INF(M).OR.J.EQ.LAT) GO TO 409 00012620
0956      C
0957      408 DO 409 M=1, NLAY          00012630
0958      IF(M.EQ.1.AND.J.GT.INF(1)) GO TO 410 00012640
0959      IF(M.EQ.1) GO TO 388          00012650
0960      IF(J.LE.INF(M-1).OR.J.GT.INF(M)) GO TO 410 00012660
0961      409 IF(J.EQ.1) GO TO 410          00012670
0962      IF(J.EQ.INF(M).OR.J.EQ.LAT) GO TO 411 00012680
0963      C
0964      410 DO 411 M=1, NLAY          00012690
0965      IF(M.EQ.1.AND.J.GT.INF(1)) GO TO 412 00012700
0966      IF(M.EQ.1) GO TO 388          00012710
0967      IF(J.LE.INF(M-1).OR.J.GT.INF(M)) GO TO 412 00012720
0968      411 IF(J.EQ.1) GO TO 412          00012730
0969      IF(J.EQ.INF(M).OR.J.EQ.LAT) GO TO 413 00012740
0970      C
0971      412 DO 413 M=1, NLAY          00012750
0972      IF(M.EQ.1.AND.J.GT.INF(1)) GO TO 414 00012760
0973      IF(M.EQ.1) GO TO 388          00012770
0974      IF(J.LE.INF(M-1).OR.J.GT.INF(M)) GO TO 414 00012780
0975      413 IF(J.EQ.1) GO TO 414          00012790
0976      IF(J.EQ.INF(M).OR.J.EQ.LAT) GO TO 415 00012800
0977      C
0978      414 DO 415 M=1, NLAY          00012810
0979      IF(M.EQ.1.AND.J.GT.INF(1)) GO TO 416 00012820
0980      IF(M.EQ.1) GO TO 388          00012830
0981      IF(J.LE.INF(M-1).OR.J.GT.INF(M)) GO TO 416 00012840
0982      415 IF(J.EQ.1) GO TO 416          00012850
0983      IF(J.EQ.INF(M).OR.J.EQ.LAT) GO TO 417 00012860
0984      C
0985      416 DO 417 M=1, NLAY          00012870
0986      IF(M.EQ.1.AND.J.GT.INF(1)) GO TO 418 00012880
0987      IF(M.EQ.1) GO TO 388          00012890
0988      IF(J.LE.INF(M-1).OR.J.GT.INF(M)) GO TO 418 00012900
0989      417 IF(J.EQ.1) GO TO 418          00012910
0990      IF(J.EQ.INF(M).OR.J.EQ.LAT) GO TO 419 00012920
0991      C
0992      418 DO 419 M=1, NLAY          00012930
0993      IF(M.EQ.1.AND.J.GT.INF(1)) GO TO 420 00012940
0994      IF(M.EQ.1) GO TO 388          00012950
0995      IF(J.LE.INF(M-1).OR.J.GT.INF(M)) GO TO 420 00012960
0996      419 IF(J.EQ.1) GO TO 420          00012970
0997      IF(J.EQ.INF(M).OR.J.EQ.LAT) GO TO 421 00012980
0998      C
0999      420 DO 421 M=1, NLAY          00012990
1000      IF(M.EQ.1.AND.J.GT.INF(1)) GO TO 422 00013000
1001      IF(M.EQ.1) GO TO 388          00013010
1002      IF(J.LE.INF(M-1).OR.J.GT.INF(M)) GO TO 422 00013020
1003      421 IF(J.EQ.1) GO TO 422          00013030
1004      IF(J.EQ.INF(M).OR.J.EQ.LAT) GO TO 423 00013040
1005      C
1006      422 DO 423 M=1, NLAY          00013050
1007      IF(M.EQ.1.AND.J.GT.INF(1)) GO TO 424 00013060
1008      IF(M.EQ.1) GO TO 388          00013070
1009      IF(J.LE.INF(M-1).OR.J.GT.INF(M)) GO TO 424 00013080
1010      423 IF(J.EQ.1) GO TO 424          00013090
1011      IF(J.EQ.INF(M).OR.J.EQ.LAT) GO TO 425 00013100
1012      C
1013      424 DO 425 M=1, NLAY          00013110
1014      IF(M.EQ.1.AND.J.GT.INF(1)) GO TO 426 00013120
1015      IF(M.EQ.1) GO TO 388          00013130
1016      IF(J.LE.INF(M-1).OR.J.GT.INF(M)) GO TO 426 00013140
1017      425 IF(J.EQ.1) GO TO 426          00013150
1018      IF(J.EQ.INF(M).OR.J.EQ.LAT) GO TO 427 00013160
1019      C
1020      426 DO 427 M=1, NLAY          00013170
1021      IF(M.EQ.1.AND.J.GT.INF(1)) GO TO 428 00013180
1022      IF(M.EQ.1) GO TO 388          00013190
1023      IF(J.LE.INF(M-1).OR.J.GT.INF(M)) GO TO 428 00013200
1024      427 IF(J.EQ.1) GO TO 428          00013210
1025      IF(J.EQ.INF(M).OR.J.EQ.LAT) GO TO 429 00013220
1026      C
1027      428 DO 429 M=1, NLAY          00013230
1028      IF(M.EQ.1.AND.J.GT.INF(1)) GO TO 430 00013240
1029      IF(M.EQ.1) GO TO 388          00013250
1030      IF(J.LE.INF(M-1).OR.J.GT.INF(M)) GO TO 430 00013260
1031      429 IF(J.EQ.1) GO TO 430          00013270
1032      IF(J.EQ.INF(M).OR.J.EQ.LAT) GO TO 431 00013280
1033      C
1034      430 DO 431 M=1, NLAY          00013290
1035      IF(M.EQ.1.AND.J.GT.INF(1)) GO TO 432 00013300
1036      IF(M.EQ.1) GO TO 388          00013310
1037      IF(J.LE.INF(M-1).OR.J.GT.INF(M)) GO TO 432 00013320
1038      431 IF(J.EQ.1) GO TO 432          00013330
1039      IF(J.EQ.INF(M).OR.J.EQ.LAT) GO TO 433 00013340
1040      C
1041      432 DO 433 M=1, NLAY          00013350
1042      IF(M.EQ.1.AND.J.GT.INF(1)) GO TO 434 00013360
1043      IF(M.EQ.1) GO TO 388          00013370
1044      IF(J.LE.INF(M-1).OR.J.GT.INF(M)) GO TO 434 00013380
1045      433 IF(J.EQ.1) GO TO 434          00013390
1046      IF(J.EQ.INF(M).OR.J.EQ.LAT) GO TO 435 00013400
1047      C
1048      434 DO 435 M=1, NLAY          00013410
1049      IF(M.EQ.1.AND.J.GT.INF(1)) GO TO 436 00013420
1050      IF(M.EQ.1) GO TO 388          00013430
1051      IF(J.LE.INF(M-1).OR.J.GT.INF(M)) GO TO 436 00013440
1052      435 IF(J.EQ.1) GO TO 436          00013450
1053      IF(J.EQ.INF(M).OR.J.EQ.LAT) GO TO 437 00013460
1054      C
1055      436 DO 437 M=1, NLAY          00013470
1056      IF(M.EQ.1.AND.J.GT.INF(1)) GO TO 438 00013480
1057      IF(M.EQ.1) GO TO 388          00013490
1058      IF(J.LE.INF(M-1).OR.J.GT.INF(M)) GO TO 438 00013500
1059      437 IF(J.EQ.1) GO TO 438          00013510
1060      IF(J.EQ.INF(M).OR.J.EQ.LAT) GO TO 439 00013520
1061      C
1062      438 DO 439 M=1, NLAY          00013530
1063      IF(M.EQ.1.AND.J.GT.INF(1)) GO TO 440 00013540
1064      IF(M.EQ.1) GO TO 388          00013550
1065      IF(J.LE.INF(M-1).OR.J.GT.INF(M)) GO TO 440 00013560
1066      439 IF(J.EQ.1) GO TO 440          00013570
1067      IF(J.EQ.INF(M).OR.J.EQ.LAT) GO TO 441 00013580
1068      C
1069      440 DO 441 M=1, NLAY          00013590
1070      IF(M.EQ.1.AND.J.GT.INF(1)) GO TO 442 00013600
1071      IF(M.EQ.1) GO TO 388          00013610
1072      IF(J.LE.INF(M-1).OR.J.GT.INF(M)) GO TO 442 00013620
1073      441 IF(J.EQ.1) GO TO 442          00013630
1074      IF(J.EQ.INF(M).OR.J.EQ.LAT) GO TO 443 00013640
1075      C
1076      442 DO 443 M=1, NLAY          00013650
1077      IF(M.EQ.1.AND.J.GT.INF(1)) GO TO 444 00013660
1078      IF(M.EQ.1) GO TO 388          00013670
1079      IF(J.LE.INF(M-1).OR.J.GT.INF(M)) GO TO 444 00013680
1080      443 IF(J.EQ.1) GO TO 444          00013690
1081      IF(J.EQ.INF(M).OR.J.EQ.LAT) GO TO 445 00013700
1082      C
1083      444 DO 445 M=1, NLAY          00013710
1084      IF(M.EQ.1.AND.J.GT.INF(1)) GO TO 446 00013720
1085      IF(M.EQ.1) GO TO 388          00013730
1086      IF(J.LE.INF(M-1).OR.J.GT.INF(M)) GO TO 446 00013740
1087      445 IF(J.EQ.1) GO TO 446          00013750
1088      IF(J.EQ.INF(M).OR.J.EQ.LAT) GO TO 447 00013760
1089      C
1090      446 DO 447 M=1, NLAY          00013770
1091      IF(M.EQ.1.AND.J.GT.INF(1)) GO TO 448 00013780
1092      IF(M.EQ.1) GO TO 388          00013790
1093      IF(J.LE.INF(M-1).OR.J.GT.INF(M)) GO TO 448 00013800
1094      447 IF(J.EQ.1) GO TO 448          00013810
1095      IF(J.EQ.INF(M).OR.J.EQ.LAT) GO TO 449 00013820
1096      C
1097      448 DO 449 M=1, NLAY          00013830
1098      IF(M.EQ.1.AND.J.GT.INF(1)) GO TO 450 00013840
1099      IF(M.EQ.1) GO TO 388          00013850
1100      IF(J.LE.INF(M-1).OR.J.GT.INF(M)) GO TO 450 00013860
1101      449 IF(J.EQ.1) GO TO 450          00013870
1102      IF(J.EQ.INF(M).OR.J.EQ.LAT) GO TO 451 00013880
1103      C
1104      450 DO 451 M=1, NLAY          00013890
1105      IF(M.EQ.1.AND.J.GT.INF(1)) GO TO 452 00013900
1106      IF(M.EQ.1) GO TO 388          00013910
1107      IF(J.LE.INF(M-1).OR.J.GT.INF(M)) GO TO 452 00013920
1108      451 IF(J.EQ.1) GO TO 452          00013930
1109      IF(J.EQ.INF(M).OR.J.EQ.LAT) GO TO 453 00013940
1110      C
1111      452 DO 453 M=1, NLAY          00013950
1112      IF(M.EQ.1.AND.J.GT.INF(1)) GO TO 454 00013960
1113      IF(M.EQ.1) GO TO 388          00013970
1114      IF(J.LE.INF(M-1).OR.J.GT.INF(M)) GO TO 454 00013980
1115      453 IF(J.EQ.1) GO TO 454          00013990
1116      IF(J.EQ.INF(M).OR.J.EQ.LAT) GO TO 455 00014000
1117      C
1118      454 DO 455 M=1, NLAY          00014010
1119      IF(M.EQ.1.AND.J.GT.INF(1)) GO TO 456 00014020
1120      IF(M.EQ.1) GO TO 388          00014030
1121      IF(J.LE.INF(M-1).OR.J.GT.INF(M)) GO TO 456 00014040
1122      455 IF(J.EQ.1) GO TO 456          00014050
1123      IF(J.EQ.INF(M).OR.J.EQ.LAT) GO TO 457 00014060
1124      C
1125      456 DO 457 M=1, NLAY          00014070
1126      IF(M.EQ.1.AND.J.GT.INF(1)) GO TO 458 00014080
1127      IF(M.EQ.1) GO TO 388          00014090
1128      IF(J.LE.INF(M-1).OR.J.GT.INF(M)) GO TO 458 00014100
1129      457 IF(J.EQ.1) GO TO 458          00014110
1130      IF(J.EQ.INF(M).OR.J.EQ.LAT) GO TO 459 00014120
1131      C
1132      458 DO 459 M=1, NLAY          00014130
1133      IF(M.EQ.1.AND.J.GT.INF(1)) GO TO 460 00014140
1134      IF(M.EQ.1) GO TO 388          00014150
1135      IF(J.LE.INF(M-1).OR.J.GT.INF(M)) GO TO 460 00014160
1136      459 IF(J.EQ.1) GO TO 460          00014170
1137      IF(J.EQ.INF(M).OR.J.EQ.LAT) GO TO 461 00014180
1138      C
1139      460 DO 461 M=1, NLAY          00014190
1140      IF(M.EQ.1.AND.J.GT.INF(1)) GO TO 462 00014200
1141      IF(M.EQ.1) GO TO 388          00014210
1142      IF(J.LE.INF(M-1).OR.J.GT.INF(M)) GO TO 462 00014220
1143      461 IF(J.EQ.1) GO TO 462          00014230
1144      IF(J.EQ.INF(M).OR.J.EQ.LAT) GO TO 463 00014240
1145      C
1146      462 DO 463 M=1, NLAY          00014250
1147      IF(M.EQ.1.AND.J.GT.INF(1)) GO TO 464 00014260
1148      IF(M.EQ.1) GO TO 388          00014270
1149      IF(J.LE.INF(M-1).OR.J.GT.INF(M)) GO TO 464 00014280
1150      463 IF(J.EQ.1) GO TO 464          00014290
1151      IF(J.EQ.INF(M).OR.J.EQ.LAT) GO TO 465 00014300
1152      C
1153      464 DO 465 M=1, NLAY          00014310
1154      IF(M.EQ.1.AND.J.GT.INF(1)) GO TO 466 00014320
1155      IF(M.EQ.1) GO TO 388          00014330
1156      IF(J.LE.INF(M-1).OR.J.GT.INF(M)) GO TO 466 00014340
1157      465 IF(J.EQ.1) GO TO 466          00014350
1158      IF(J.EQ.INF(M).OR.J.EQ.LAT) GO TO 467 00014360
1159      C
1160      466 DO 467 M=1, NLAY          00014370
1161      IF(M.EQ.1.AND.J.GT.INF(1)) GO TO 468 00014380
1162      IF(M.EQ.1) GO TO 388          00014390
1163      IF(J.LE.INF(M-1).OR.J.GT.INF(M)) GO TO 468 00014400
1164      467 IF(J.EQ.1) GO TO 468          00014410
1165      IF(J.EQ.INF(M).OR.J.EQ.LAT) GO TO 469 00014420
1166      C
1167      468 DO 469 M=1, NLAY          00014430
1168      IF(M.EQ.1.AND.J.GT.INF(1)) GO TO 470 00014440
1169      IF(M.EQ.1) GO TO 388          00014450
1170      IF(J.LE.INF(M-1).OR.J.GT.INF(M)) GO TO 470 00014460
1171      469 IF(J.EQ.1) GO TO 470          00014470
1172      IF(J.EQ.INF(M).OR.J.EQ.LAT) GO TO 471 00014480
1173      C
1174      470 DO 471 M=1, NLAY          00014490
1175      IF(M.EQ.1.AND.J.GT.INF(1)) GO TO 472 00014500
1176      IF(M.EQ.1) GO TO 388          00014510
1177      IF(J.LE.INF(M-1).OR.J.GT.INF(M)) GO TO 472 00014520
1178      471 IF(J.EQ.1) GO TO 472          00014530
1179      IF(J.EQ.INF(M).OR.J.EQ.LAT) GO TO 473 00014540
1180      C
1181      472 DO 473 M=1, NLAY          00014550
1182      IF(M.EQ.1.AND.J.GT.INF(1)) GO TO 474 00014560
1183      IF(M.EQ.1) GO TO 388          00014570
1184      IF(J.LE.INF(M-1).OR.J.GT.INF(M)) GO TO 474 00014580
1185      473 IF(J.EQ.1) GO TO 474          00014590
1186      IF(J.EQ.INF(M).OR.J.EQ.LAT) GO TO 475 00014600
1187      C
1188      474 DO 475 M=1, NLAY          00014610
1189      IF(M.EQ.1.AND.J.GT.INF(1)) GO TO 476 00014620
1190      IF(M.EQ.1) GO TO 388          00014630
1191      IF(J.LE.INF(M-1).OR.J.GT.INF(M)) GO TO 476 00014640
1192      475 IF(J.EQ.1) GO TO 476          00014650
1193      IF(J.EQ.INF(M).OR.J.EQ.LAT) GO TO 477 00014660
1194      C
1195      476 DO 477 M=1, NLAY          00014670
1196      IF(M.EQ.1.AND.J.GT.INF(1)) GO TO 478 00014680
1197      IF(M.EQ.1) GO TO 388          00014690
1198      IF(J.LE.INF(M-1).OR.J.GT.INF(M)) GO TO 478 00014700
1199      477 IF(J.EQ.1) GO TO 478          00014710
1200      IF(J.EQ.INF(M).OR.J.EQ.LAT) GO TO 479 00014720
1201      C
1202      478 DO 479 M=1, NLAY          00014730
1203      IF(M.EQ.1.AND.J.GT.INF(1)) GO TO 480 00014740
1204      IF(M.EQ.1) GO TO 388          00014750
1205      IF(J.LE.INF(M-1).OR.J.GT.INF(M)) GO TO 480 00014760
1206      479 IF(J.EQ.1) GO TO 480          00014770
1207      IF(J.EQ.INF(M).OR.J.EQ.LAT) GO TO 481 00014780
1208      C
1209      480 DO 481 M=1, NLAY          00014790
1210      IF(M.EQ.1.AND.J.GT.INF(1))
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0878      SYZJ = HR*(4.*X(JJ6+1)-3.*X(JJ1+1)-X(JJ12+1))      00012190
0879      SXZ = HR*(4.*X(JJ6)-3.*X(JJ1)-X(JJ12))      00012200
0880      GO TO 391      00012210
C
0881      390 SZ = HR*(3.*X(JJ1+2)+X(JJ10+2)-4.*X(JJ8+2))      00012220
0882      SYZJ = HR*(3.*X(JJ1+1)+X(JJ10+1)-4.*X(JJ8+1))      00012230
0883      SXZ = HR*(3.*X(JJ1)+X(JJ10)-4.*X(JJ8))      00012240
C
0884      391 SYZ = SYZI + SYZJ      00012250
C
C      CALCULATION OF THE STRESS (T)      00012260
C
0885      TX = C11(M)*SX + C12(M)*SY + C13(M)*SZ + C16(M)*SXY      00012270
0886      TY = C12(M)*SX + C22(M)*SY + C23(M)*SZ + C26(M)*SXY      00012280
0887      TZ = C13(M)*SX + C23(M)*SY + C33(M)*SZ + C36(M)*SXY      00012290
C
0888      TYZ = C44(M)*SYZ + C45(M)*SXZ      00012300
0889      TXZ = C45(M)*SYZ + C55(M)*SXZ      00012310
0890      TXY = C16(M)*SX + C26(M)*SY + C36(M)*SZ + C66(M)*SXY      00012320
C
0891      WRITE(6,672) NODE, TX, TY, TZ, TYZ, TXZ, TXY, SY, SZ, SYZ, SXZ, SXY 00012330
0892      WRITE(6,397) SX      00012340
C
C      STRESS AND STRAINS JUST ABOVE AN INTERFACE      00012350
C
0893      IF(J.NE.INF(M).OR.J.EQ.LAT) GO TO 392      00012360
0894      P = M+1      00012370
0895      SZ = HR*(4.*X(JJ6+2)-3.*X(JJ1+2)-X(JJ12+2))      00012380
0896      SYZJ = HR*(4.*X(JJ6+1)-3.*X(JJ1+1)-X(JJ12+1))      00012390
0897      SXZ = HR*(4.*X(JJ6)-3.*X(JJ1)-X(JJ12))      00012400
0898      SYZ = SYZI + SYZJ      00012410
C
0899      TX = C11(P)*SX + C12(P)*SY + C13(P)*SZ + C16(P)*SXY      00012420
0900      TY = C12(P)*SX + C22(P)*SY + C23(P)*SZ + C26(P)*SXY      00012430
0901      TZ = C13(P)*SX + C23(P)*SY + C33(P)*SZ + C36(P)*SXY      00012440
C
0902      TYZ = C44(P)*SYZ + C45(P)*SXZ      00012450
0903      TXZ = C45(P)*SYZ + C55(P)*SXZ      00012460
0904      TXY = C16(P)*SX + C26(P)*SY + C36(P)*SZ + C66(P)*SXY      00012470
C
0905      WRITE(6,672) NODE, TX, TY, TZ, TYZ, TXZ, TXY, SY, SZ, SYZ, SXZ, SXY 00012480
C
0906      392 CONTINUE      00012490
0907      398 CONTINUE      00012500
0908      WRITE(6,652)      00012510
0909      399 CONTINUE      00012520
0910      9000 CONTINUE      00012530
C
C
0911      397 FORMAT(14X,1P1E11.3/)      00012540
0912      600 FORMAT(1H1, 44X, 44H*** UNIFORM BENDING OF A LAMINATED PLATE ***) 00012550
0913      601 FORMAT(5I10)      00012560
C
C
C *****      00012570
C
C      FORMATS      00012580
C
C *****      00012590
C
C *****      00012600
C
C *****      00012610
C
C *****      00012620
C
C *****      00012630
C
C *****      00012640
C
C *****      00012650
C
C *****      00012660
C
C *****      00012670
C
C *****      00012680
C
C *****      00012690
C
C *****      00012700
C
C *****      00012710
C
C *****      00012720
C
C *****      00012730
C
C *****      00012740
C
C *****      00012750
C
C *****      00012760

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0914      602 FORMAT(//// 5X, 18H*** INPUT DATA *** //)
          1     18X, 'NUMBER OF LAYERS IN CROSS SECTION, NLAY =', 14 // 00012770
          2     18X, 'NUMBER OF NODES ON VERTICAL AXIS, LAT =', 14 // 00012780
          3     18X, 'NUMBER OF NODES ON HORIZONTAL AXIS, LAW =', 14 // 00012790
          4     18X, 37HCHANGE IN MESH WIDTH (FSW1) AT I = , 14 // 00012800
          5     18X, 37HCHANGE IN MESH WIDTH (FSW2) AT I = , 14 // 00012810
          6     18X, 37HMESH WIDTH MAGNIFICATION FACTOR, K = , 14 // 00012820
          C
0915      603 FORMAT(8G12.5)
          C
0916      604 FORMAT(1H1, 55X, 21H*** MATERIAL DATA *** // 2X, 5HLAYER, 7X, 00012830
          1     3HE11, 9X, 3HE22, 9X, 3HE33, 9X, 3HE12, 9X, 3HE13, 9X, 00012840
          2     3HE23, 8X, 4HNU12, 4X, 4HNU13, 4X, 4HNU23 // ) 00012850
          C
0917      605 FORMAT(3X, I2, 6X, 2PE10.3, 2(2X, 1PE10.3), 3(2X, 0PE10.3), 00012860
          1     3(3X, F5.2) / ) 00012870
          C
0918      606 FORMAT(10G10.3)
          C
0919      607 FORMAT(/// 18X, 26HFINE SIMULATION WIDTH, H = ,F8.5) 00012880
          C
0920      608 FORMAT(// 18X, 9HLAYER NO., 2X, I3, 5X, 17HINTERFACE AT J = ,I3) 00012890
          C
0921      611 FORMAT(// 45X, 41H*** COEFFICIENTS OF THERMAL EXPANSION ***, //)
          1     1X, 5HLAYER, 8X, 5HTHETA, 12X, 3HAL1, 12X, 3HAL2, 12X, 00012900
          2     3HAL3, 12X, 3HAL6, 12X, 4HAL1P, 11X, 4HAL2P, 11X, 4HAL3P 00012910
          3     // )
          C
0922      613 FORMAT(// 53X, 26H*** STIFFNESS MATRICES *** // 1X,
          1     11HLAYER/THETA, 21X, 12HX-Y-Z MATRIX, 44X, 00012920
          2     18HX-Y-Z PRIME MATRIX // ) 00012930
          C
0923      614 FORMAT(2X, I2, 9X, F5.1, 5X, 7(5X, E10.3)) 00012940
          C
0924      620 FORMAT(2X, I2, 5X, 1P12E10.3 // 19X, 5E10.3, 10X, 5E10.3 // 29X, 00012950
          1     4E10.3, 20X, 4E10.3 // 1X, 0PF5.1, 33X, 1P3E10.3, 30X, 00012960
          2     3E10.3 // 49X, 2E10.3, 40X, 2E10.3 // 59X, E10.3, 50X, 00012970
          3     E10.3 // )
          C
0925      650 FORMAT(1H1 // 10X, '*** GRID POINT DISPLACEMENT FUNCTIONS ***' // 00012980
          1     16X, 5H NODE, 5X, 14HU-DISPLACEMENT, 6X, 14HV-DISPLACEMENT, 00012990
          2     6X, 14HW-DISPLACEMENT // ) 00013000
          C
0926      651 FORMAT(10X, I10, 3E20.6 // )
          652 FORMAT(// 12H ***** // ) 00013010
          653 FORMAT(// 10X, 12H ***** // ) 00013020
          C
0929      670 FORMAT(1H1, 10X, 77H*** OUTPUT STRESSES AND STRAINS FOR A MAXIMUM 00013030
          1LONGITUDINAL BENDING STRAIN OF , F6.0, 22H MICRO-INCHES/INCH AND / 00013040
          2 48X, 40H AN APPLIED AXIAL EXTENSIONAL STRAIN OF , F6.0, 00013050
          3 19H MICRO-INCHES/INCH. // 10X, 'NOTE: INTERFACE NODES ARE REPEAT 00013060
          4ED WITH VALUES GIVEN BELOW AND ABOVE THE INTERFACE RESPECTIVELY.' 00013070
          5 // )
          C
0930      671 FORMAT(1X,5HNODE , 5X, 5HSIG-X, 6X, 5HSIG-Y, 6X, 5HSIG-Z, 6X, 00013080
          1     6HTAU-YZ, 5X, 6HTAU-XZ, 5X, 6HTAU-XY, 5X, 5HEPS-Y, 6X, 00013090
          2     5HEPS-Z, 6X, 6HEPS-YZ, 5X, 6HEPS-XZ, 5X, 6HEPS-XY / 17X, 00013100
          3     5HEPS-X // ) 00013110
          C
0931      672 FORMAT(1X, I3, 4X, 1P11E11.3 /)
          C
0932      STOP 00013120
0933      END 00013130

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MATCON

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0001      SUBROUTINE MATCON                               00013400
          C                                              00013410
          C *****CALCULATION OF LAMINATE LOAD CONSTANTS FOR A FULL CROSS SECTION***** 00013420
          C                                              00013430
          C *****CALCULATION OF LAMINATE LOAD CONSTANTS FOR A FULL CROSS SECTION***** 00013440
          C                                              00013450
          C *****CALCULATION OF LAMINATE LOAD CONSTANTS FOR A FULL CROSS SECTION***** 00013460
          C                                              00013470
          C THIS SUBROUTINE IS GOOD FOR BENDING OF AN ARBITRARILY LAID UP      00013471
          C LAMINATE WHICH IS SYMMETRIC OR NONSYMMETRIC ABOUT THE MIDPLANE.      00013472
          C                                              00013480
          C THE CONSTANTS ARE  C2 = INVERSE BENDING RADIUS      00013490
          C C3E = APPLIED UNIFORM EXTENSIONAL STRAIN      00013500
          C C3 = EXTENSIONAL COUPLING DUE TO BENDING PLUS C3E      00013510
          C C4 = IN-PLANE SHEAR COUPLING      00013520
          C                                              00013530
          C BU OCCURS IN THE FCTN. U(Y,Z)      00013540
          C BV AND DV OCCUR IN THE FCTN. V(Y,Z)      00013550
          C                                              00013560
          C SXMAX (EFFECTIVELY THE LOAD INPUT) IS A MAXIMUM STRAIN      00013570
          C                                              00013580
0002      INTEGER ORDER                               00013590
          C                                              00013600
0003      COMMON /MC/ C11(6),C12(6),C16(6),C22(6),C26(6),C66(6),C13(6),      00013610
          1      C23(6),C36(6),C44(6),C45(6),C55(6),C33(6),AL1(6),AL2(6),      00013620
          2      AL3(6),AL6(6),C2,C3,C3E,C4,BU,DU,BV,DV,H,SXMAX,NLAY,INF(6) 00013630
          C                                              00013640
0004      DIMENSION A(3,3), B(3,3), D(3,3), QM(3,3)      00013650
          C                                              00013660
0005      DOUBLE PRECISION A, B, D      00013670
          C                                              00013680
0006      ORDER = 3      00013690
          C                                              00013700
0007      LAY = INF(1)-1      00013710
0008      HL = H*FLOAT(LAY)      00013720
0009      HL2 = HL**2/2.      00013730
0010      HL3 = HL**3/3.      00013740
0011      RN = FLOAT(NLAY)      00013750
0012      RN2 = RN**2      00013760
          C                                              00013770
0013      DO 20 I=1,3      00013780
0014      DO 20 J=1,3      00013790
0015      A(I,J) = 0.D0      00013800
0016      B(I,J) = 0.D0      00013810
0017      D(I,J) = 0.D0      00013820
0018      20 CONTINUE      00013830
          C                                              00013840
0019      DO 30 I=1,3      00013850
0020      DO 30 J=1,3      00013860
0021      DO 30 M=1,NLAY      00013870
0022      QM(1,1) = C11(M)-C13(M)*C13(M)/C33(M)      00013880
0023      QM(1,2) = C12(M)-C13(M)*C23(M)/C33(M)      00013890
0024      QM(1,3) = C16(M)-C13(M)*C36(M)/C33(M)      00013900
0025      QM(2,1) = QM(1,2)      00013910
0026      QM(2,2) = C22(M)-C23(M)*C23(M)/C33(M)      00013920
0027      QM(2,3) = C26(M)-C23(M)*C36(M)/C33(M)      00013930
0028      QM(3,1) = QM(1,3)      00013940
0029      QM(3,2) = QM(2,3)      00013950

```

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 0030 QM(3,3) = C66(M)-C36(M)\*C36(M)/C33(M) 00013960  
 C 00013970  
 C NOTE THAT THE SUBSCRIPT 3 IN QM REPLACES A 6 IN STANDARD NOTATION. 00013980  
 C THE SAME IS TRUE BELOW IN A(I,J), B(I,J), D(I,J), ETC. 00013990  
 C 00014000  
 0031 M1 = 2\*M-1 00014010  
 0032 M2 = 3\*M\*(M-1)+1 00014020  
 C 00014030  
 0033 A(I,J) = A(I,J) + HL\*QM(I,J) 00014040  
 0034 B(I,J) = B(I,J) + HL2\*QM(I,J)\*(M1-RN) 00014050  
 0035 D(I,J) = D(I,J) + HL3\*QM(I,J)\*(M2-1.5\*RN\*M1+.75\*RN2) 00014060  
 0036 30.CONTINUE 00014070  
 C 00014080  
 C INVERT (A). STORE IN (A). 00014090  
 C 00014100  
 0037 CALL MATIN4 (A,ORDER) 00014110  
 C 00014120  
 C MULTIPLY (A) INVERSE \* (B). STORE IN A. 00014130  
 C 00014140  
 0038 CALL MAMULT (A,B,ORDER,A) 00014150  
 C 00014160  
 C MULTIPLY (B) \* (A) INVERSE \* (B). STORE IN B. 00014170  
 C 00014180  
 0039 CALL MAMULT (B,A,ORDER,B) 00014190  
 C 00014200  
 0040 DO 40 I=1,3 00014210  
 0041 DO 40 J=1,3 00014220  
 0042 A(I,J) = -1.\*A(I,J) 00014230  
 0043 D(I,J) = D(I,J) - B(I,J) 00014240  
 0044 40 CONTINUE 00014250  
 C 00014260  
 C INVERT NEW MATRIX (D). THE RESULT IS D-PRIME. STORE IN D. 00014270  
 C 00014280  
 0045 CALL MATIN4 (D,ORDER) 00014290  
 C 00014300  
 C MULTIPLY -(A) INVERSE \* B \* D-PRIME WHICH YIELDS B-PRIME. STORE IN B. 00014310  
 C 00014320  
 0046 CALL MAMULT (A,D,ORDER,B) 00014330  
 C 00014340  
 C DETERMINE THE LOAD CONSTANTS. MINUS C2 IMPLIES A SMILING PLATE. 00014350  
 C 00014360  
 0047 ZMAX = RN\*HL/2. 00014370  
 0048 C2 = -D(1,1)\*SMAX/(B(1,1) +D(1,1)\*ZMAX) 00014380  
 0049 RATIO = C2/D(1,1) 00014390  
 C 00014400  
 0050 C3 = B(1,1)\*RATIO + C3E 00014410  
 0051 C4 = .5\*D(1,3)\*RATIO 00014420  
 0052 BU = B(3,1)\*RATIO 00014430  
 0053 BV = B(2,1)\*RATIO 00014440  
 0054 DV = D(1,2)\*RATIO 00014450  
 C RATIO = -RATIO 00014460  
 0055 WRITE(6,50) 00014470  
 0056 50 FORMAT(//// 48X, 35H\*\*\* THE LAMINATE LOAD CONSTANTS \*\*\* // ) 00014480  
 0057 WRITE(6,60) C2, C3, C4, BU, BV, DV, RATIO 00014490  
 0058 60 FORMAT(' C2 = ', 1PE10.3, 4X, 'C3 = ', E10.3, 4X, 'C4 = ', E10.3, 00014500  
 1 4X, 'BU = ', E10.3, 4X, 'BV = ', E10.3, 4X, 'DV = ', E10.3, 00014510  
 2 4X, 'MT = ', E10.3 ) 00014520  
 0059 RETURN 00014530  
 0060 END 00014540

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MATCON

DATE = 75082

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\*OPTIONS IN EFFECT\* NOTERM,NOID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP,NOTEST  
\*OPTIONS IN EFFECT\* NAME = MATCON , LINECNT = 60  
\*STATISTICS\* SOURCE STATEMENTS = 60,PROGRAM SIZE = 2060  
\*STATISTICS\* NO DIAGNOSTICS GENERATED

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MAMULT

DATE = 75082

19/49/20

0001 SUBROUTINE MAMULT(B,C,N,A) 00014550  
C 00014551  
C MAMULT POSTMULTIPLIES MATRIX (B) BY MATRIX (C) AND STORES THE 00014552  
C RESULT IN MATRIX (A) WHERE N IS THE ORDER OF THE MATRICES. 00014553  
C 00014554  
0002 DOUBLE PRECISION A,B,C,SUM 00014560  
0003 DIMENSION A(N,N), B(N,N), C(N,N) 00014570  
0004 DO 1 I=1,N 00014580  
0005 DO 1 J=1,N 00014590  
0006 SUM = 0. 00014600  
0007 DO 2 K=1,N 00014610  
0008 SUM = SUM + B(I,K)\*C(K,J) 00014620  
0009 2 CONTINUE 00014630  
0010 A(I,J) = SUM 00014640  
0011 1 CONTINUE 00014650  
0012 RETURN 00014660  
0013 END 00014670

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MAMULT

DATE = 75082

19/49/20

\*OPTIONS IN EFFECT\* NOTERM,NOID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP,NOTEST  
\*OPTIONS IN EFFECT\* NAME = MAMULT , LINECNT = 60  
\*STATISTICS\* SOURCE STATEMENTS = 13,PROGRAM SIZE = 702  
\*STATISTICS\* NO DIAGNOSTICS GENERATED

FORTRAN IV G1 RELEASE 2.0

MATIN4

DATE = 75082

19/49/20

0001 SUBROUTINE MATIN4(ARRAY,N) 00014680  
C 00014681  
C MATIN4 INVERTS THE MATRIX (ARRAY) WHICH IS OF ORDER N. 00014682  
C 00014683  
0002 DIMENSION ARRAY(N,N) 00014690  
0003 DOUBLE PRECISION ARRAY 00014700  
0004 DO 604 I=1,N 00014710  
0005 STORE = ARRAY(I,I) 00014720  
0006 ARRAY(I,I) = 1. 00014730  
0007 DO 601 J=1,N 00014740  
0008 601 ARRAY(I,J) = ARRAY(I,J)/STORE 00014750  
0009 DD 604 K=1,N 00014760  
0010 IF(K-1)602,604,602 00014770  
0011 602 STORE = ARRAY(K,I) 00014780  
0012 ARRAY(K,I) = 0. 00014790  
0013 DO 603 J=1,N 00014800  
0014 603 ARRAY(K,J) = ARRAY(K,J) - STORE\*ARRAY(I,J) 00014810  
0015 604 CONTINUE 00014820  
0016 RETURN 00014830  
0017 END 00014840

```

0001      SUBROUTINE TRMSTR(A,N,ND,NLD,NRD,NED,D,R,E)          00014850
C
C      TRMSTR IS THE SUBROUTINE TRIMSS WITH MATRIX A TRANSPOSED. 00014860
C      THE SIMULTANEOUS SOLUTIONS IS GAUSSIAN ELIMINATION,      00014870
C      MODIFIED TO TAKE ADVANTAGE OF THE REDUCED MATRIX. THE      00014880
C      ROUTINE ALSO USES PARTIAL PIVOTING TO REDUCE ROUND OFF ERROR. 00014890
C
C      INPUT
C          1 A      FIRST LOCATION OF COEFFICIENT MATRIX, I.E. A(1,1). 00014900
C          THE BAND ELEMENTS IN EACH ROW MUST BE LEFT                00014930
C          JUSTIFIED AND EXTEND TO THE RIGHT M PLACES               00014940
C          (M=MIN(N,NLD+NRD+1). IF IN ANY PARTICULAR ROW            00014950
C          THERE ARE ONLY K BAND ELEMENTS AND K IS LESS              00014960
C          THAN M, THEN THE M-K RIGHT MOST ELEMENTS OF THAT          00014970
C          ROW WILL BE SET TO ZERO. THE ROW WHOSE LEFT              00014980
C          MOST COLUMN IN THE FULL BLOWN MATRIX CONTAINS           00014990
C          A NON-ZERO ELEMENT MUST BE THE FIRST ROW OF THE          00015000
C          REDUCED MATRIX AND ETC. THE COLUMN TO THE                00015010
C          IMMEDIATE RIGHT OF THE REDUCED MATRIX (FORMED AS        00015020
C          ABOVE) MUST CONTAIN THE RIGHT HAND SIDE OF THE          00015030
C          EQUATION SET IN QUESTION. IT SHOULD NOW BE            00015040
C          OBVIOUS THAT AN N X N+1 FULL BLOWN SYSTEM WOULD        00015050
C          BE REDUCED BY THE ABOVE METHOD TO AN N X M+1            00015060
C          SYSTEM.                                              00015070
C          2 N      NUMBER OF SIMULTANEOUS EQUATIONS TO BE SOLVED. 00015080
C          3 ND      VARIABLE DIMENSION INTEGER. MUST BE EQUAL TO 00015090
C          ROW DIMENSION OF A IN CALLING PROGRAM.                 00015100
C          4 NLD     MAXIMUM NUMBER OF BAND ELEMENTS TO THE LEFT 00015110
C          OF PRINCIPAL DIAGONAL IN ANY ROW OF SYSTEM TO          00015120
C          BE DETERMINED.                                         00015130
C          5 NRD     MAXIMUM NUMBER OF BAND ELEMENTS TO THE RIGHT 00015140
C          OF PRINCIPAL DIAGONAL IN ANY ROW OF SYSTEM TO          00015150
C          BE DETERMINED.                                         00015160
C          6 NED     NED=MIN(N,NLD+NRD+1)                           00015170
C
C      OUTPUT
C          1 A      THE FIRST COLUMN OF A CONTAINS THE SOLUTION 00015180
C          VECTOR.                                              00015190
C          2 D      CONTAINS DETERMINANT OF A.                      00015200
C          3 R      CONTAINS RANK OF A.                         00015220
C          4 E      E=0., SOLUTION O.K. E=1., A SINGULAR.        00015230
C          E=2., SOLUTION ATTEMPTED, BUT A ILL CONDITIONED        00015240
C          OR SINGULAR. IN THIS CASE SOLUTIONS SHOULD BE        00015250
C          CHECKED TO ASSURE VALIDITY.                           00015260
C
C      SUBROUTINE TRMSTR(A,N,ND,NLD,NRD,NED,D,R,E)          00015280
0002      DIMENSION A(ND,1)                                     00015290
0003      DOUBLE PRECISION A,D,Y,W,S                         00015300
0004      X1 = 1.                                              00015310
0005      L1 = 1.                                              00015320
0006      E=0.                                                 00015330
0007      R = 0.                                                 00015340
0008      D=1.                                                 00015350
0009      ND1=NED+1                                         00015360
0010      M=NLD                                              00015370
0011      NM1=N-1                                           00015380
0012      DO 1 I=1,NM1                                         00015390
0013      IF(I.GT.(N-NLD))M=M-1                           00015400
0014      NN=I+M-1                                         00015410
0015      DO 2 II=I,NN                                         00015420

```

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TRMSTR

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```

0016      IF(DABS(A(1,I)).GE.DABS(A(1,II+1))) GO TO 2          00015430
0017      D=-D          00015440
0018      DO 3 J=1,ND1          00015450
0019      Y=A(1,I)          00015460
0020      A(J,I)=A(J,II+1)          00015470
0021      3 A(J,II+1)=Y          00015480
0022      2 CONTINUE          00015490
0023      C      D=D*A(1,I)          00015500
0024      IF(A(1,I) .EQ. 0.) GO TO 10          00015510
0025      GO TO (5,13),L1          00015520
0026      13 IF(DABS(DABS((X1-A(1,I))/X1)-1.).LT.1.E-07) E=2. 00015530
0027      X1 = A(1,I)          00015540
0028      5 R = R + 1.          00015550
0029      L1 = 2          00015560
0030      DO 4 J=2,ND1          00015570
0031      4 A(J,I)=A(J,I) / A(1,I)          00015580
0032      K=I+1          00015590
0033      NN=I+M          00015600
0034      DO 1 II=K,NN          00015610
0035      W=A(1,II)          00015620
0036      DO 6 J=1,NED          00015630
0037      6 A(J,II)=A(J+1,II)-A(J+1,I)*W          00015640
0038      A(ND1,II)=A(NED,II)          00015650
0039      1 A(NED,II)=0.          00015660
0040      IF(A(1,N).EQ.0.)GO TO 10          00015670
0041      IF(DABS(DABS((X1-A(1,N))/X1)-1.).LT.1.E-07) E=2. 00015680
0042      9 R = R + 1.          00015690
0043      A(1,N)=A(ND1,N)/A(1,N)          00015700
0044      K=NM1          00015710
0045      NN=2          00015720
0046      8 IF(NN.GT.NED)NN=NED          00015730
0047      J=K+1          00015740
0048      S=0.          00015750
0049      DO 7 I=2,NN          00015760
0050      7 J=J+1          00015770
0051      A(1,K)=A(ND1,K)-S          00015780
0052      NN=NN+1          00015790
0053      K=K-1          00015800
0054      IF(K.NE.0)GO TO 8          00015810
0055      RETURN          00015820
0056      10 E=1.          00015830
0057      RETURN          00015840
0058      END          00015850
                                         00015860

```

FORTRAN IV G1 RELEASE 2.0

TRMSTR

DATE = 75007

08/16/07

```

*OPTIONS IN EFFECT*  NOTERM,NOID,EBCDIC,SOURCE,NOLIST,NOCKECK,LOAD,NOMAP,NOTEST
*OPTIONS IN EFFECT*  NAME = TRMSTR , LINECNT = 60
*STATISTICS*  SOURCE STATEMENTS = 58,PROGRAM SIZE = 2294
*STATISTICS*  NO DIAGNOSTICS GENERATED

```

FORTRAN IV G1 RELEASE 2.0 RITE DATE = 75007 08/16/07

```
0001      SUBROUTINE RITE(IDUM,NR,NC,MR,MC,A)          00015870
0002      DOUBLE PRECISION A                          00015880
0003      DIMENSION A(MR,MC)                         00015890
0004      IPRINT= 12                                00015900
0005      IF(IDUM,NE.1) IPRINT= 30                  00015910
0006      IPR= IPRINT-1                            00015920
0007      DO 35 K=1,NC,IPR                         00015930
0008      MAX= K+IPR                                00015940
0009      IF(MAX.GT.NC) MAX=NC                     00015950
0010      IF(K,NE.1) WRITE(6,103)                  00015960
0011      45 WRITE(6,102) (I,I=K,MAX)             00015970
0012      DO 40 J=1,NR                           00015980
0013      40 WRITE(6,105) J,(A(J,I),I=K,MAX)      00015990
0014      35 CONTINUE                                00016000
0015      RETURN                                     00016010
0016      101 FORMAT(6X,30I4)                      00016020
0017      102 FORMAT(6X,12I10)                     00016030
0018      103 FORMAT('1')                         00016040
0019      104 FORMAT(' ',I5,30I4)                  00016050
0020      105 FORMAT(' ',I5,12G10.3)             00016060
0021      END                                     00016070
```

FORTRAN IV G1 RELEASE 2.0 RITE DATE = 75007 08/16/07

```
*OPTIONS IN EFFECT* NOTERM,NOID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP,NOTEST
*OPTIONS IN EFFECT* NAME = RITE      , LINECNT =      60
*STATISTICS*   SOURCE STATEMENTS =      21,PROGRAM SIZE =      864
*STATISTICS*   NO DIAGNOSTICS GENERATED
*STATISTICS*   NO DIAGNOSTICS THIS STEP
```

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